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Analysis of turbulent combustion in inert porous media ☆

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ABSTRACT

The objective of this paper is to present an extension of a simplified reaction kinetics model that, combined with a thermo-mechanical closure, entails a full-generalized turbulent combustion model for flow in porous media. In this model, one explicitly considers the intra-pore levels of turbulent kinetic energy. Transport equations are written in their time-and-volume-averaged form and a volume-based statistical turbulence model is applied to simulate turbulence generation due to the porous matrix. The rate of fuel consumption is described by an Arrhenius expression involving the product of the fuel and oxidant mass fractions. These mass fractions are double decomposed in time and space and, after applying simultaneous time-and-volume integration operations to them, distinct terms arise, which are here associated with the mechanisms of dispersion and turbulence. Modeling of these extra terms remains an open question and the derivations herein might motivate further development of models for turbulent combustion in porous media.

1. Introduction

Analysis and simulation of turbulent combustion has attracted researchers for decades for their countless applications in science and engineering. Studies on free flame flows have been presented for a wide range of systems, including basic research [1–4] and numerical simulations [5–9], involving, among many configurations and cases, swirling flows [10–13] and applications spanning from fire simulation [14–18] to equipment development [19–24].

In addition to studies on free flame flows, the advantages of having a combustion process inside an inert porous matrix are today well recognized [25–28]. A variety of applications of efficient radiant porous burners can be encountered in the power and process industries, requiring proper mathematical tools for reliable design and analysis of such efficient engineering equipment.

The literature already covers a wide range of studies on combustion in porous media [29–40], including recent reviews on burning of gases [41] and liquids [42] in such burners. Hsu et al. [43] points out some of its benefits including higher burning speed and volumetric energy release rates, higher combustion stability and the ability to burn gases of a low energy content. Driven by this motivation, the effects on porous ceramics inserts have been investigated in Peard et al. [44], among others.

The majority of the publications on combustion in porous media consider the flow to remain in the laminar regime while undergoing chemical exothermic reaction. However, recent awareness of the importance of treating intra-pore turbulence has motivated authors in developing models for turbulent flow in porous media, with [45] and without combustion [46]. Accordingly, turbulence modeling of combustion within inert porous media has been conducted by Lim and Matthews [45] on the basis of an extension of the standard k-ε model of Jones and Launder [47]. In [45] the ε equation was discarded in lieu of prescription of an appropriate length scale. Work on direct simulation of laminar premixed flames has also been reported in Sahraoui and Kaviany [48].

In addition, non-reactive turbulence flow in porous media has been the subject of several studies [49–51], including applications of flows though porous baffles [52], channels with porous inserts [53] and buoyant flows [54]. In this series of papers, a concept called double-decomposition was proposed [55], in which variables were decomposed simultaneously in time and space. Also, intra-pore turbulence was accounted for in all transport equations, but only non-reactive flow has been previously investigated in [49–55].

The objective of this paper is to apply the double-decomposition concept, previously proposed for non-reacting flows, to a simple combustion closure for turbulent flow through porous media. By that, a full turbulent combustion model is presented, in which the mechanisms of dispersion and turbulence are incorporated in the consumption rates of the fuel. Derivations herein might contribute to the development of more elaborated models for combustion in porous materials.

2. Macroscopic thermo-mechanical model

As mentioned, the thermo-mechanical model here employed is based on the double-decomposition concept [49,55], which has been also described in detail in a book [51]). In that work, transport equations are volume averaged according to the Volume Averaging

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Nomenclature

Latin characters

A	Pre-exponential factor
c_F	Forchheimer coefficient
c_p	Specific heat
$D=[\nabla\mathbf{u}+(\nabla\mathbf{u})^T]/2$	Deformation rate tensor
D_ℓ	Diffusion coefficient of species ℓ
D_{diff}	Macroscopic diffusion coefficient
D_{disp}	Dispersion tensor due to dispersion
$D_{disp,t}$	Dispersion tensor due to turbulene
f_2	Damping function
f_μ	Damping function
D_{eff}	Effective dispersion
K	Permeability
k_f	Fluid thermal conductivity
k_s	Solid thermal conductivity
$\mathbf{K}_{eff}$	Effective Conductivity tensor
m_ℓ	Mass fraction of species ℓ
Pr	Prandtl number
S_{fu}	Rate of fuel consumption
T	Temperature
$\mathbf{u}$	Microscopic velocity
$\mathbf{u}_D$	Darcy or superficial velocity (volume average of $\mathbf{u}$)

Greek characters

α	Thermal diffusivity
β_r	Extinction coefficient
ΔV	Representative elementary volume
ΔV_f	Fluid volume inside ΔV
ΔH	Heat of combustion
μ	Dynamic viscosity
ν	Kinematic viscosity
ρ	Density
φ	$\varphi = {}^{\Delta}V_f/_{\Delta}V$, Porosity
ψ	Excess air-to-fuel ratio

Special characters

φ	General variable
$\langle\varphi\rangle^i$	Intrinsic average
$\langle\varphi\rangle^v$	Volume average
$^i\varphi$	Spatial deviation
$\overline{\varphi}$	Time average
$^i\varphi$	Spatial deviation
$\|\varphi\|$	Absolute value (Abs)
$\boldsymbol{\varphi}$	Vectorial general variable
$()_{s,f}$	solid/fluid
$()_{eff}$	Effective value, $\phi\varphi_f+(1-\phi)\varphi_s$
$()_\varphi$	Macroscopic value
$()_{fu}$	Fuel
$()_{ox}$	Oxygen

Theorem [56–58] in addition of using time decomposition of flow variables followed by standard time-averaging procedure for treating turbulence. As the entire equation set is already fully available in open literature, these equations will be reproduced here and details about their derivations can be obtained in the aforementioned references. Essentially, in all the above-mentioned work the flow variables are decomposed in a volume mean and a deviation (classical porous media analysis) in addition of being also decomposed in a time-mean and a fluctuation (classical turbulent flow treatment). Because mathematical details and proofs of such concept are available in a number of worldwide available papers in the literature, they are not repeated here. These final equations in their steady-state form are:

2.1. Macroscopic continuity equation

$$\nabla.\rho_f\overline{\mathbf{u}}_D = 0 \tag{1}$$

where, $\overline{\mathbf{u}}_D$ is the average surface velocity (also known as seepage, superficial, filter or Darcy velocity) and ρ_f is the fluid density. Eq. (1) represents the macroscopic continuity equation for the gas.

2.2. Macroscopic momentum equation

$$\nabla\cdot\left(\rho_f\frac{\overline{\mathbf{u}}_D\,\overline{\mathbf{u}}_D}{\phi}\right) = -\nabla\left(\phi\langle\overline{p}\rangle^i\right)+\mu\nabla^2\overline{\mathbf{u}}_D+\nabla\cdot\left(-\rho_f\phi\langle\overline{\mathbf{u}'\,\mathbf{u}'}\rangle^i\right) + \phi\,\rho_f\,\mathbf{g}-\left[\frac{\mu\phi}{K}\overline{\mathbf{u}}_D+\frac{c_F\phi\,\rho_f\mid\overline{\mathbf{u}}_D\mid\overline{\mathbf{u}}_D}{\sqrt{K}}\right] \tag{2}$$

where the last two terms in Eq. (2), represent the Darcy and Forchheimer contributions. The symbol K is the porous medium permeability, $c_F=0.55$ is the form drag coefficient, $\langle p\rangle^i$ is the intrinsic (fluid phase averaged) pressure of the fluid, μ represents the fluid viscosity and ϕ is the porosity of the porous medium.

Turbulence is handled via a macroscopic $k-\varepsilon$ model given by,

$$\nabla\cdot(\rho_f\overline{\mathbf{u}}_D\langle k\rangle^i) = \nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_k}\right)\nabla\left(\varphi\langle k\rangle^i\right)\right]-\rho_f\langle\overline{\mathbf{u}'\,\mathbf{u}'}\rangle^i:\nabla\overline{\mathbf{u}}_D + c_k\rho_f\frac{\phi\langle k\rangle^i\mid\overline{\mathbf{u}}_D\mid}{\sqrt{K}}-\rho_f\phi\langle\varepsilon\rangle^i \tag{3}$$

$$\nabla\cdot(\rho_f\overline{\mathbf{u}}_D\langle\varepsilon\rangle^i) = \nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_\varepsilon}\right)\nabla\left(\phi\langle\varepsilon\rangle^i\right)\right]+c_1\left(-\rho_f\langle\overline{\mathbf{u}'\,\mathbf{u}'}\rangle^i:\nabla\overline{\mathbf{u}}_D\right)\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i} + c_2\,c_k\rho_f\frac{\phi\langle\varepsilon\rangle^i\mid\overline{\mathbf{u}}_D\mid}{\sqrt{K}}-c_2\,\rho_f\phi\frac{\langle\varepsilon\rangle^{i2}}{\langle k\rangle^i} \tag{4}$$

where

$$-\rho_f\phi\langle\overline{\mathbf{u}'\,\mathbf{u}'}\rangle^i = \mu_{t_\phi}2\langle\overline{\mathbf{D}}\rangle^v-\frac{2}{3}\phi\,\rho_f\langle k\rangle^i\mathbf{I} \tag{5}$$

and

$$\mu_{t_\varphi} = \rho_f c_\mu\frac{\langle k\rangle^{i2}}{\langle\varepsilon\rangle^i}. \tag{6}$$

Details on the derivation of the above equations can be found in [51].

2.3. Macroscopic energy equations

Macroscopic energy equations are obtained for both fluid and solid phases by also applying time and volume average operators to the instantaneous local equations [59]. As in the flow case, volume integration is performed over a Representative Elementary Volume (REV). After including the heat released due to the combustion reaction, one gets for both phases:

$$\text{Gas}:\nabla\cdot(\rho_f c_{pf}\mathbf{u}_D\langle\overline{T}_f\rangle^i) = \nabla\cdot\left\{\mathbf{K}_{eff,f}\cdot\nabla\langle\overline{T}_f\rangle^i\right\}+h_i a_i\left(\langle\overline{T}_s\rangle^i-\langle\overline{T}_f\rangle^i\right)+\phi\Delta H S_{fu}, \tag{7}$$

$$\text{Solid}:0 = \nabla\cdot\left\{\mathbf{K}_{eff,s}\cdot\nabla\langle\overline{T}_s\rangle^i\right\}-h_i a_i\left(\langle\overline{T}_s\rangle^i-\langle\overline{T}_f\rangle^i\right), \tag{8}$$

where, $a_i = A_i/\Delta V$ is the interfacial area per unit volume, h_i is the film coefficient for interfacial transport, $\mathbf{K}_{eff,f}$ and $\mathbf{K}_{eff,s}$ are the effective conductivity tensors for fluid and solid, respectively, given by,

$$\mathbf{K}_{eff,f} = \left\{ \overbrace{\phi k_f}^{conduction} \right\} \mathbf{I} + \underbrace{\mathbf{K}_{f,s}}_{local\ conduction} + \underbrace{\mathbf{K}_{disp}}_{dispersion} + \underbrace{\mathbf{K}_t + \mathbf{K}_{disp,t}}_{turbulence} \tag{9}$$

$$\mathbf{K}_{eff,s} = \left\{ \overbrace{(1-\phi)[k_s}^{conduction} + \frac{\overbrace{16\sigma(\langle \overline{T} \rangle^i)^3}^{radiation}}{3\beta_r}] \right\} \mathbf{I} + \underbrace{\mathbf{K}_{s,f}}_{local\ conduction} . \tag{10}$$

In Eqs. (7)–(10), $\mathbf{I}$ is the unit tensor, ΔH is the heat of combustion, β_r is the extinction coefficient, σ is the Stephan-Boltzman constant [5.66961×10^{-8} W/m^{2}K^4] and S_{fu} is the rate of fuel consumption, to be commented below. All mechanisms contributing to heat transfer within the medium, together with turbulence and radiation, are included in order to compare their effect on temperature distribution. Further, such distinct contributions of various mechanisms are the outcome of the application of gradient type diffusion models, in the form (see [59] for details).

$$\text{Turbulent heat flux} : -\left(\rho c_p\right)_f \left(\phi \, \overline{\langle \mathbf{u}' \rangle^i \langle T_f' \rangle^i} \right) = \mathbf{K}_t \cdot \nabla \langle \overline{T}_f \rangle^i . \tag{11}$$

$$\text{Thermal dispersion} : -\left(\rho c_p\right)_f \left(\phi \, \langle {}^i\overline{\mathbf{u}}^i \overline{T}_f \rangle^i \right) = \mathbf{K}_{disp} \cdot \nabla \langle \overline{T}_f \rangle^i . \tag{12}$$

$$\text{Turbulent thermal dispersion} : -\left(\rho c_p\right)_f \left(\phi \, \langle \overline{{}^i\mathbf{u}'^i T_f'} \rangle^i \right) = \mathbf{K}_{disp,t} \cdot \nabla \langle \overline{T}_f \rangle^i . \tag{13}$$

$$\text{Local conduction} : \begin{aligned} \nabla \cdot \left[\frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i k_f \overline{T}_f dA \right] &= \mathbf{K}_{f,s} \cdot \nabla \langle \overline{T}_s \rangle^i \\ \nabla \cdot \left[\frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i k_s \overline{T}_s dA \right] &= \mathbf{K}_{s,f} \cdot \nabla \langle \overline{T}_f \rangle^i . \end{aligned} \tag{14}$$

In Eqs. (7) and (8) the heat transferred between the two phases was modeled by means of a film coefficient h_i. A numerical correlation for the interfacial convective heat transfer coefficient was proposed by Kuwahara et al. [60] for laminar flow as:

$$\frac{h_i D}{k_f} = \left(1 + \frac{4(1-\phi)}{\phi}\right) + \frac{1}{2}(1-)^{1/2} \mathrm{Re}_D \mathrm{Pr}^{1/3}, \text{valid for } 0.2 < \phi < 0.9. \tag{15}$$

For turbulent flow, the following expression was proposed in Saito and de Lemos [59]:

$$\frac{h_i D}{k_f} = 0.08 \left(\frac{Re_D}{\phi}\right)^{0.8} Pr^{1/3}; \text{for} 1.0 \times 10^4 < \frac{Re_D}{\phi} < 2.0 \times 10^7, \text{valid for } 0.2 < \varphi < 0.9, \tag{16}$$

2.4. Macroscopic mass transport

Transport equation for the fuel reads,

$$\nabla \cdot (\rho_f \overline{\mathbf{u}}_D \langle \overline{m}_{fu} \rangle^i) = \nabla \cdot \rho_f \mathbf{D}_{eff} \cdot \nabla (\phi \langle \overline{m}_{fu} \rangle^i) - \phi S_{fu} \tag{17}$$

where $\langle \overline{m}_{fu} \rangle^i$ is the mass fraction for the fuel. The effective mass transport tensor, $\mathbf{D}_{eff}$, is defined as:

$$\mathbf{D}_{eff} = \underbrace{\mathbf{D}_{disp}}_{dispersion} + \overbrace{\mathbf{D}_{diff}}^{diffusion} + \underbrace{\mathbf{D}_t + \mathbf{D}_{disp,t}}_{turbulence} = \mathbf{D}_{disp} + \frac{1}{\rho_f}\left(\frac{\mu_b}{Sc_\ell} + \frac{\mu_{t_b}}{Sc_{\ell,t}}\right)\mathbf{I} = \mathbf{D}_{disp} + \frac{1}{\rho_f}\left(\frac{\mu_{b,eff}}{Sc_{\ell,eff}}\right)\mathbf{I} \tag{18}$$

where Sc_ℓ and $Sc_{\ell,t}$ are the laminar and turbulent Schmidt numbers for species ℓ, respectively, and "*eff*" denotes an effective value. The dispersion tensor is defined such that,

$$-\rho_f \phi \langle {}^i\overline{\mathbf{u}}^i \overline{m}_{fu} \rangle^i = \rho_f \mathbf{D}_{disp} \cdot \nabla (\phi \langle \overline{m}_{fu} \rangle^i). \tag{19}$$

3. Macroscopic combustion model

3.1. Simple chemistry

In this work, for simplicity, the chemical exothermic reaction is assumed to be instantaneous and to occur in a single step, kinetic-controlled, which, for combustion of a mixture air/methane, is given by the chemical reaction [36–38],

$$CH_4 + 2(1+\Psi)(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 2\Psi O_2 + 7.52(1+\Psi)N_2. \tag{20}$$

For *N*-heptane, a similar equation reads [38],

$$C_7H_{16} + 11(1+\Psi)(O_2 + 3.76N_2) \rightarrow 7CO_2 + 8H_2O + 11\Psi O_2 + 41.36(1+\Psi)N_2. \tag{21}$$

And for Octane, we have,

$$C_8H_{18} + 12.5(1+\Psi)(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 12.5\Psi O_2 + 47(1+\Psi)N_2 \tag{22}$$

where Ψ is the excess air in the reactant stream at the inlet of the porous foam. For the stoichiometric ratio, $\Psi = 0$. In all of these equations, the reaction is then assumed to be kinetically controlled and occurring infinitely fast. A general expression for them can be derived as,

$$\begin{aligned} C_nH_{2m} + (n + \tfrac{m}{2})(1+\Psi)(O_2 + 3.76N_2) \rightarrow \\ nCO_2 + mH_2O + (n + \tfrac{m}{2})\Psi O_2 + (n + \tfrac{m}{2})3.76(1+\Psi)N_2 \end{aligned} \tag{23}$$

where the coefficients n and m can be found in Table 1. Eq. (23) is here assumed to hold for the particular examples given in the table.

The local instantaneous rate of fuel consumption over the total volume (fluid plus solid) was determined by a one step Arrhenius reaction [61,62] given by,

$$S_{fu} = \rho_f^a A m_{fu}^b m_{ox}^c \, e^{-E/R\langle \overline{T} \rangle^i} \tag{24}$$

where m_{fu} and m_{ox} are the local instantaneous mass fractions for the fuel and oxidant, respectively, and coefficients a, b and c depend on the particular reaction [62]. For simplicity in presenting the ideas below, we assume here $a = 2, b = c = 1$, which corresponds to burning

Table 1
Coefficients in the general combustion Eq. (23).

Gas	n	m	$(n+m/2)$	$(n+m/2)\times 3.76$
Methane	1	2	2	7.52
N-heptane	7	8	11	41.36
Octane	8	9	12.5	47

of a mixture of methane and air [36–38,63]. Also, in Eq. (24) A is the pre-exponential factor and E is the activation energy, where numerical values for these parameters depend on the fuel considered [61].

Density ρ_f in the above equations is determined from the perfect gas equation for a mixture of perfect gases:

$$\rho_f = \frac{P_o}{RT_f \sum_{1}^{\ell} \frac{m_\ell}{M_\ell}} \tag{25}$$

where P_o is the absolute pressure, R is the universal gas constant [8.134 J/(mol K)] and M_ℓ is the molecular weight of species ℓ.

3.2. Double-decomposition of variables

Macroscopic transport equations for turbulent flow in a porous medium are obtained through the simultaneous application of time and volume average operators over a generic fluid property φ. Such concepts are defined as [56–58].

$$\langle\varphi\rangle^i = \frac{1}{\Delta V_f}\int_{\Delta V_f} \varphi dV;\ \langle\varphi\rangle^v = \phi\langle\varphi\rangle^i;\ \phi = \frac{\Delta V_f}{\Delta V},\ \text{with}\ \varphi = \langle\varphi\rangle^i + {}^i\varphi \tag{26}$$

$$\overline{\varphi} = \frac{1}{\Delta t}\int_t^{t+\Delta t}\varphi dt,\ \text{with}\ \phi = \overline{\varphi} + \varphi' \tag{27}$$

where ΔV_f is the volume of the fluid contained in a Representative Elementary Volume (REV) ΔV, intrinsic average and volume average are represented, respectively, by $\langle\rangle^i$ and $\langle\rangle^v$. Also, due to the definition of average we have,

$$\overline{\varphi'} = 0 \tag{28}$$

and

$$\langle{}^i\varphi\rangle^i = 0. \tag{29}$$

The double decomposition idea, introduced and fully described in [55], combines Eqs. (26) and (27) and can be summarized as:

$$\overline{\langle\varphi\rangle^i} = \langle\overline{\varphi}\rangle^i;\ {}^i\overline{\varphi} = \overline{{}^i\varphi};\ \langle\varphi'\rangle^i = \langle\varphi\rangle^{i\prime} \tag{30}$$

and,

$$\left.\begin{array}{l}\varphi' = \langle\varphi'\rangle^i + {}^i\varphi' \\ {}^i\varphi = \overline{{}^i\varphi} + {}^i\varphi'\end{array}\right\}\ \text{where}\ \ {}^i\varphi' = \varphi' - \langle\varphi'\rangle^i = {}^i\varphi - \overline{{}^i\varphi}. \tag{31}$$

Therefore, the quantity φ can be expressed by either,

$$\varphi = \overline{\langle\varphi\rangle^i} + \langle\varphi\rangle^{i\prime} + \overline{{}^i\varphi} + {}^i\varphi' \tag{32}$$

or

$$\varphi = \langle\overline{\varphi}\rangle^i + {}^i\overline{\varphi} + \langle\varphi'\rangle^i + {}^i\varphi'. \tag{33}$$

The term ${}^i\varphi'$ can be viewed as either the temporal fluctuation of the spatial deviation or the spatial deviation of the temporal fluctuation of the quantity φ.

3.3. Macroscopic fuel consumption rates

In order to derive macroscopic equations also for the simple combustion model presented above, we can take Eq. (24) with $a=2$, $b=c=1$ and note that the rate of fuel consumption is dictated by product of two local instantaneous values, m_{fu} and m_{ox}, which represent local instantaneous mass fractions for the fuel and oxygen, respectively. Now, if we apply to each one of them the decomposition (32), or its Eq. (33), we get,

$$m_{fu} = \langle\overline{m}_{fu}\rangle^i + {}^i\overline{m}_{fu} + \langle m'_{fu}\rangle^i + {}^i m'_{fu} \tag{34}$$

$$m_{ox} = \langle\overline{m}_{ox}\rangle^i + {}^i\overline{m}_{ox} + \langle m'_{ox}\rangle^i + {}^i m'_{ox}. \tag{35}$$

For the sake of simplicity and manipulation, looking in Eq. (24) at only the product of the mass fractions ($m_{fu}m_{ox}$), and applying the decompositions (34) and (35), we get,

$$\begin{aligned} m_{fu}m_{ox} = {} & \langle\overline{m}_{fu}\rangle^i\langle\overline{m}_{ox}\rangle^i + {}^i\overline{m}_{fu}\langle\overline{m}_{ox}\rangle^i + \langle m'_{fu}\rangle^i\langle\overline{m}_{ox}\rangle^i + {}^i m'_{fu}\langle\overline{m}_{ox}\rangle^i \\ & + \langle\overline{m}_{fu}\rangle^i{}^i\overline{m}_{ox} + {}^i\overline{m}_{fu}\,{}^i\overline{m}_{ox} + \langle m^i_{fu}\rangle^i{}^i\overline{m}_{ox} + {}^i m'_{fu}\,{}^i\overline{m}_{ox} \\ & + \langle\overline{m}_{fu}\rangle^i\langle m'_{ox}\rangle^i + {}^i\overline{m}_{fu}\langle m'_{ox}\rangle^i + \langle m'_{fu}\rangle^i\langle m'_{ox}\rangle^i + {}^i m'_{fu}\langle m'_{ox}\rangle^i \\ & + \langle\overline{m}_{fu}\rangle^i{}^i m'_{ox} + {}^i\overline{m}_{fu}\,{}^i m'_{ox} + \langle m'_{fu}\rangle^i{}^i m'_{ox} + {}^i m'_{fu}\,{}^i m'_{ox}. \end{aligned} \tag{36}$$

Applying the volume-average operator (26) to the instantaneous local product (36), we get,

$$\begin{aligned} \langle m_{fu}m_{ox}\rangle^i = {} & \langle\langle\overline{m}_{fu}\rangle^i\langle\overline{m}_{ox}\rangle^i\rangle^i + \langle{}^i\overline{m}_{fu}\langle\overline{m}_{ox}\rangle^i\rangle^i + \langle\langle m'_{fu}\rangle^i\langle\overline{m}_{ox}\rangle^i\rangle^i \\ & + \langle{}^i m'_{fu}\langle\overline{m}_{ox}\rangle^i\rangle^i + \langle\langle\overline{m}_{fu}\rangle^i{}^i\overline{m}_{ox}\rangle^i + \langle{}^i\overline{m}_{fu}\,{}^i\overline{m}_{ox}\rangle^i \\ & + \langle\langle m'_{fu}\rangle^i{}^i\overline{m}_{ox}\rangle^i + \langle{}^i m'_{fu}\,{}^i\overline{m}_{ox}\rangle^i + \langle\langle\overline{m}_{fu}\rangle^i\langle m'_{ox}\rangle^i\rangle^i \\ & + \langle{}^i\overline{m}_{fu}\langle m'_{ox}\rangle^i\rangle^i + \langle\langle m'_{fu}\rangle^i\langle m'_{ox}\rangle^i\rangle^i + \langle{}^i m'_{fu}\langle m'_{ox}\rangle^i\rangle^i \\ & + \langle\langle\overline{m}_{fu}\rangle^i{}^i m'_{ox}\rangle^i + \langle{}^i\overline{m}_{fu}\,{}^i m'_{ox}\rangle^i + \langle\langle m'_{fu}\rangle^i{}^i m'_{ox}\rangle^i \\ & + \langle{}^i m'_{fu}\,{}^i m'_{ox}\rangle^i. \end{aligned} \tag{37}$$

Now, looking back at condition (29), all terms containing only one deviation factor in Eq. (37) will vanish, such that,

$$\begin{aligned} \langle m_{fu}m_{ox}\rangle^i = {} & \langle\langle\overline{m}_{fu}\rangle^i\langle\overline{m}_{ox}\rangle^i\rangle^i + \langle{}^i\overline{m}_{fu}\langle\overline{m}_{ox}\rangle^i\rangle^i + \langle\langle m'_{fu}\rangle^i\langle\overline{m}_{ox}\rangle^i\rangle^i + \langle{}^i m'_{fu}\langle\overline{m}_{ox}\rangle^i\rangle^i \\ & + \langle\langle\overline{m}_{fu}\rangle^i{}^i\overline{m}_{ox}\rangle^i + \langle{}^i\overline{m}_{fu}\,{}^i\overline{m}_{ox}\rangle^i + \langle\langle m'_{fu}\rangle^i{}^i\overline{m}_{ox}\rangle^i + \langle{}^i m'_{fu}\,{}^i\overline{m}_{ox}\rangle^i \\ & + \langle\langle\overline{m}_{fu}\rangle^i\langle m'_{ox}\rangle^i\rangle^i + \langle{}^i\overline{m}_{fu}\langle m'_{ox}\rangle^i\rangle^i + \langle\langle m'_{fu}\rangle^i\langle m'_{ox}\rangle^i\rangle^i + \langle{}^i m'_{fu}\langle m'_{ox}\rangle^i\rangle^i \\ & + \langle\langle\overline{m}_{fu}\rangle^i{}^i m'_{ox}\rangle^i + \langle{}^i\overline{m}_{fu}\,{}^i m'_{ox}\rangle^i + \langle\langle m'_{fu}\rangle^i{}^i m'_{ox}\rangle^i + \langle{}^i m'_{fu}\,{}^i m'_{ox}\rangle^i \end{aligned} \tag{38}$$

and the following equation is left as,

$$\begin{aligned} \langle m_{fu}m_{ox}\rangle^i = {} & \langle\overline{m}_{fu}\rangle^i\langle\overline{m}_{ox}\rangle^i + \langle m'_{fu}\rangle^i\langle\overline{m}_{ox}\rangle^i + \langle{}^i\overline{m}_{fu}\,{}^i\overline{m}_{ox}\rangle^i \\ & + \langle{}^i m'_{fu}\,{}^i\overline{m}_{ox}\rangle^i + \langle\overline{m}_{fu}\rangle^i\langle m'_{ox}\rangle^i + \langle m'_{fu}\rangle^i\langle m'_{ox}\rangle^i \\ & + \langle{}^i\overline{m}_{fu}\,{}^i m'_{ox}\rangle^i + \langle{}^i m'_{fu}\,{}^i m'_{ox}\rangle^i. \end{aligned} \tag{39}$$

Another form to write Eq. (39), using the equivalences shown in Eq. (30), is

$$\begin{aligned} \langle m_{fu}m_{ox}\rangle^i = {} & \overline{\langle m_{fu}\rangle^i\langle m_{ox}\rangle^i} + \langle m_{fu}\rangle^{i\prime}\overline{\langle m_{ox}\rangle^i} + \langle\overline{{}^i m_{fu}}\,\overline{{}^i m_{ox}}\rangle^i + \langle{}^i m'_{fu}\,\overline{{}^i m_{ox}}\rangle^i \\ & + \overline{\langle m_{fu}\rangle^i}\langle m_{ox}\rangle^{i\prime} + \langle m_{fu}\rangle^{i\prime}\langle m_{ox}\rangle^{i\prime} + \langle\overline{{}^i m_{fu}}\,{}^i m'_{ox}\rangle^i \\ & + \langle{}^i m'_{fu}\,{}^i m'_{ox}\rangle^i. \end{aligned} \tag{40}$$

If we now apply the time-averaging operator over Eq. (40) and note that, due to condition (28), all terms containing only one time fluctuation factor vanish, such that,

$$\overline{\langle m_{fu} m_{ox}\rangle^i} = \overline{\langle m_{fu}\rangle^i \langle m_{ox}\rangle^i} + \overline{\langle m_{fu}\rangle^{i\prime} \langle m_{ox}\rangle^{i\prime}} + \overline{\langle {}^i m_{fu}\, {}^i m_{ox}\rangle^i} + \overline{\langle {}^i m'_{fu}\, {}^i m_{ox}\rangle^i} + \overline{\langle m_{fu}\rangle^i \langle m_{ox}\rangle^{i\prime}} + \overline{\langle m_{fu}\rangle^{i\prime} \langle m_{ox}\rangle^{i\prime}} + \overline{\langle {}^i m_{fu}\, {}^i m'_{ox}\rangle^i} + \overline{\langle {}^i m'_{fu}\, {}^i m'_{ox}\rangle^i} \tag{41}$$

we get the following time-and-volume averaged expression after dropping all null values,

$$\langle \overline{m_{fu} m_{ox}^i}\rangle = \langle \overline{m_{fu}}\rangle^i \langle \overline{m_{ox}}\rangle^i + \langle {}^i\overline{m_{fu}}\ {}^i\overline{m_{ox}}\rangle^i + \langle \overline{m_{fu}}\rangle^{i\prime} \langle m_{ox}\rangle^{i\prime} + \overline{\langle {}^i m'_{fu}\, {}^i m'_{ox}\rangle^i}. \tag{42}$$

Again, we can make use of an alternative representation for the same terms in Eq. (42) when looking at equivalences (30), we get,

$$\langle \overline{m_{fu} m_{ox}}\rangle^i = \langle \overline{m}_{fu}\rangle^i \langle \overline{m}_{ox}\rangle^i + \langle {}^i\overline{m}_{fu}\, {}^i\overline{m}_{ox}\rangle^i + \overline{\langle m'_{fu}\rangle^i \langle m'_{ox}\rangle^i} + \overline{\langle {}^i m'_{fu}\, {}^i m'_{ox}\rangle^i}. \tag{43}$$

Including now the full decomposition Eq. (43) back into the expression for S_{fu}, Eq. (24), we have,

$$\begin{aligned}\langle \overline{S_{fu}}\rangle^i &= \rho_f^2 A \langle \overline{m_{fu} m_{ox}}\rangle^i e^{-E/R\langle \overline{T}\rangle^i} \\ &= \rho_f^2 A \left(\underbrace{\langle \overline{m}_{fu}\rangle^i \overline{m}_{ox}\rangle^i}_{\text{I}} + \underbrace{\langle {}^i\overline{m}_{fu}\, {}^i\overline{m}_{ox}\rangle^i}_{\text{II}} + \underbrace{\overline{\langle m'_{fu}\rangle^i \langle m'_{ox}\rangle^i}}_{\text{III}} + \underbrace{\overline{\langle {}^i m'_{fu}\, {}^i m'_{ox}\rangle^i}}_{\text{IV}} \right) e^{-E/R\langle \overline{T}\rangle^i}.\end{aligned} \tag{44}$$

The four term on the right-hand-side of Eq. (44), multiplied by the parameter $\rho_f^2 A e^{-E/R\langle \overline{T}\rangle^i}$, can be physically interpreted as

I *Reaction rate* due to volume-and-time averaged values of fuel and oxidant mass fractions. This is the standard rate of reaction commonly employed in the literature [36–38].

II *Dispersive reaction rate* due to deviation of mean time-mean fuel and oxidant mass fractions. This rate occurs even if the flow is laminar and is due to fact that both mass fractions present a deviation about their volume-averaged values.

III *Turbulent reaction rate* due to time-fluctuation of volume-averaged values of fuel and oxidant mass flow rates, and represents an additional fuel consumption due to the fact that inside a representative elementary volume (REV), the volume-averaged mass fraction of both oxygen and fuel fluctuate with time, giving rise to a non-null time correlation.

IV *Turbulent dispersive reaction rate* due to simultaneous time fluctuations and volume deviations of both values of fuel and oxidant mass flow rates.

In light of Eq. (31), terms II and III in Eq. (44), can be recombined to form,

$$\overline{\langle m'_{fu}\rangle^i \langle m'_{ox}\rangle^i} + \overline{\langle {}^i m'_{fu}\, {}^i m'_{ox}\rangle^i} = \overline{\langle m'_{fu} m'_{ox}\rangle^i} \tag{45}$$

giving,

$$S_{fu,\varphi}^t = \rho_f^2 A \overline{\langle m'_{fu} m'_{ox}\rangle^i}\, e^{-E/R\langle \overline{T}\rangle^i} \tag{46}$$

which can be seen as the overall effect of turbulence on the fuel consumption rate. Likewise, the dispersive component reads,

$$S_{fu,\varphi}^{disp} = \rho_f^2 A \langle {}^i\overline{m}_{fu}\, {}^i\overline{m}_{ox}\rangle^i\, e^{-E/R\langle \overline{T}\rangle^i} \tag{47}$$

and for the first term in Eq. (44),

$$S_{fu,\varphi} = \rho_f^2 A \langle \overline{m}_{fu}\rangle^i \langle \overline{m}_{ox}\rangle^i\, e^{-E/R\langle \overline{T}\rangle^i} \tag{48}$$

giving finally

$$\langle \overline{S_{fu}}\rangle^i = S_{fu,\varphi} + S_{fu,\varphi}^{disp} + S_{fu,\varphi}^t \tag{49}$$

Models for Eqs. (46) and (47) and evaluation of their relative values when compared to Eq. (48) remains an open question and shall be the subject of further investigation. They might be related to physically controlled mechanisms associated with the full reaction rate Eq. (44).

4. Conclusions

This paper presents a proposal for a full two-energy equation allowing for turbulent combustion in an inert porous media. Fuel consumption rate is expressed by the kinetic controlled one-step Arrenious expression, which contains the product of two values, namely the mass fraction of the fuel and of the oxidant. The double-decomposition concept is applied to these both mass fractions giving rise to distinct terms, which could be associated with the mechanics of dispersion and turbulence in porous media. Modeling of these extra terms remains an open question. The derivations herein might shed some light on the overall developing of models for turbulent combustion in porous media.

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