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Turbulent kinetic energy in a moving porous bed ☆

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ABSTRACT

This paper presents a set of transport equations for solving problems involving turbulent flow in a moving bed reactor. The reactor is seen as a porous matrix with a moving solid phase. Equations are time-and-volume averaged and the solid phase is considered to have an imposed constant velocity. Additional drag terms appearing in the momentum equation are assumed to be a function of the relative velocity between the fluid and solid phase. Turbulence equations are influenced by the speed of the solid phase in relation to that of the flowing fluid. Results show the decrease of turbulent kinetic energy levels as the solid speed approaches the speed of the moving bed.

1. Introduction

When analyzing turbulent flow in porous media, there are many situations of practical relevance in which the porous substrate moves along with the flow, usually with a different velocity than that of the working fluid. Several manufacturing processes deal with such configuration and applied computations can be found in the literature [1–4]. Biomass pelletization and preparation for energy production may also consider systems having a moving porous bed [5,6]. Therefore, the ability to realistic model such systems is of great advantage to a number of materials, food and energy production processes.

Accordingly, a turbulence model for flow in a fixed and rigid porous media has been proposed [7,8], which is today fully documented and available in the open literature [9]. However, in all work presented in [9], no movement of the solid phase was considered. The purpose of this contribution is to extend the previous work on turbulence in porous media, exploring now configurations that consider the movement of the solid material.

2. Macroscopic model for fixed bed

A macroscopic form of the governing equations is obtained by taking the volumetric average of the entire equation set. In this development, the porous medium is considered to be rigid, fixed and saturated by the incompressible fluid. As mentioned, derivation of this equation set is already available in the literature [7–9] so that details need not to be repeated here. Nevertheless, for the sake of completeness, transport equations in their final modeled form are here presented.

The macroscopic continuity equation is given by,

$$\nabla\cdot\bar{\mathbf{u}}_D=0 \tag{1}$$

where the Dupuit–Forchheimer relationship, $\bar{\mathbf{u}}_D=\phi\langle\bar{\mathbf{u}}\rangle^i$, has been used and $\langle\bar{\mathbf{u}}\rangle^i$ identifies the intrinsic (fluid) average of the local velocity vector $\bar{\mathbf{u}}$ [10]. Eq. (1) represents the macroscopic continuity equation for an incompressible fluid in a rigid porous medium [11].

Further, the macroscopic time-mean Navier–Stokes (NS) equation for an incompressible fluid with constant properties can be written as (see [7]) for details),

$$\rho\nabla\cdot\left(\frac{\bar{\mathbf{u}}_D\bar{\mathbf{u}}_D}{\phi}\right)=-\nabla\left(\phi\langle\bar{p}\rangle^i\right)+\mu\nabla^2\bar{\mathbf{u}}_D+\nabla\cdot\left(-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i\right)-\left[\frac{\mu\phi}{K}\bar{\mathbf{u}}_D+\frac{c_F\phi\rho|\bar{\mathbf{u}}_D|\bar{\mathbf{u}}_D}{\sqrt{K}}\right] \tag{2}$$

where μ is the fluid dynamic viscosity, K is the permeability, c_F is the Forchheimer coefficient and $-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i$ is the Macroscopic Reynolds Stress Tensor (MRST), modeled as:

$$-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i=\mu_{t_\phi}2\langle\bar{\mathbf{D}}\rangle^v-\frac{2}{3}\phi\rho\langle k\rangle^i I \tag{3}$$

Also,

$$\langle\bar{\mathbf{D}}\rangle^v=\frac{1}{2}\left[\nabla\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)+\left[\nabla\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)\right]^T\right] \tag{4}$$

is the macroscopic deformation tensor, $\langle k\rangle^i$ is the intrinsic average for k and μ_{t_ϕ} is the macroscopic turbulent viscosity, which is modeled here similarly to the case of clear fluid flow. A proposal for μ_{t_ϕ} was presented in [7] as,

$$\mu_{t_\phi}=\rho c_\mu\langle k\rangle^{i^2}/\langle\varepsilon\rangle^i \tag{5}$$

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Nomenclature

c_F	Forchheimer coefficient in Eq. (2)
c's	Constants in Eqs. (5), (7), and (8)
$\mathbf{D}$	Deformation rate tensor, $\mathbf{D}=[\nabla\mathbf{u}+(\nabla\mathbf{u})^T]\,/\,2$
G^i	Production rate of $\langle k\rangle^i$ due to the porous matrix, $G^i=c_k\rho\phi\langle k\rangle^i\lvert\bar{\mathbf{u}}_D\rvert/\sqrt{K}$
H	Distance between channel walls
k	Turbulent kinetic energy per unit mass, $k=\overline{\mathbf{u}'\cdot\mathbf{u}'}/2$
$\langle k\rangle^v$	Volume (fluid + solid) average of k
$\langle k\rangle^i$	Intrinsic (fluid) average of k
K	Permeability
L	Channel length
p	Thermodynamic pressure
$\langle p\rangle^i$	Intrinsic (fluid) average of pressure p
P^i	Production rate of k due to mean gradients of $\bar{\mathbf{u}}_D$, $P^i=-\rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i:\nabla\bar{\mathbf{u}}_D$
$\bar{\mathbf{u}}$	Microscopic time-averaged velocity vector
$\langle\bar{\mathbf{u}}\rangle^i$	Intrinsic (fluid) average of $\bar{\mathbf{u}}$
$\bar{\mathbf{u}}_D$	Darcy velocity vector, $\bar{\mathbf{u}}_D=\phi\langle\bar{\mathbf{u}}\rangle^i$
$\bar{\mathbf{u}}_{\text{rel}}$	Relative velocity based on total volume, $\bar{\mathbf{u}}_{\text{rel}}=\bar{\mathbf{u}}_D-\mathbf{u}_S$
$\bar{\mathbf{u}}^{\gamma}_{\text{rel}}$	Relative velocity based on phase volume, $\bar{\mathbf{u}}^{\gamma}_{\text{rel}}=\langle\bar{\mathbf{u}}\rangle^i-\langle\mathbf{u}\rangle^s$
x, y	Cartesian coordinates
Greek	
μ	Fluid dynamic viscosity
μ_t	Turbulent viscosity
μ_{t_ϕ}	Macroscopic turbulent viscosity
ε	Dissipation rate of k, $\varepsilon=\mu\overline{\nabla\mathbf{u}':(\nabla\mathbf{u}')^T}/\rho$
$\langle\varepsilon\rangle^i$	Intrinsic (fluid) average of ε
ρ	Density
ϕ	Porosity
γ	Phase identifier
η, ξ	Generalized coordinates

For a fixed bed, a final form of Eq. (2) reads, after incorporating the models given by Eqs. (3), (4), and (5),

$$\rho\left[\nabla\cdot\left(\frac{\bar{\mathbf{u}}_D\bar{\mathbf{u}}_D}{\phi}\right)\right]=-\nabla\left(\phi\langle\bar{p}\rangle^i\right)+\mu\nabla^2\bar{\mathbf{u}}_D+\nabla\cdot\left(-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i\right)-\left[\frac{\mu\phi}{K}\bar{\mathbf{u}}_D+\frac{c_F\phi\rho\lvert\bar{\mathbf{u}}_D\rvert\bar{\mathbf{u}}_D}{\sqrt{K}}\right], \tag{6}$$

where the last two term in Eq. (6) are known as the Darcy and the Forchheimer drags. These terms represent the viscous and net pressure forces felt by the fluid after passing through the porous bed.

2.1. Macroscopic equations for turbulence

Transport equations for $\langle k\rangle^i=\langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2$ and $\langle\varepsilon\rangle^i=\mu\langle\overline{\nabla\mathbf{u}':(\nabla\mathbf{u}')^T}\rangle^i/\rho$ in their so-called High Reynolds Number form are presented in [7] as:

$$\rho\nabla\cdot\left(\bar{\mathbf{u}}_D\langle k\rangle^i\right)=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_k}\right)\nabla\left(\phi\langle k\rangle^i\right)\right]+P^i+G^i-\rho\phi\langle e\rangle^i \tag{7}$$

$$\rho\nabla.\left(\bar{\mathbf{u}}_D\langle\varepsilon\rangle^i\right)=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_\varepsilon}\right)\nabla\left(\phi\langle\varepsilon\rangle^i\right)\right]+c_1P^i\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i}+c_2\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i}\left(G^i-\rho\phi\langle e\rangle^i\right) \tag{8}$$

where the c's are constants, $P^i=-\rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i:\nabla\bar{\mathbf{u}}_D$ is the production rate of $\langle k\rangle^i$ due to gradients of $\bar{\mathbf{u}}_D$ and $G^i=c_k\rho\phi\langle k\rangle^i\lvert\bar{\mathbf{u}}_D\rvert/\sqrt{K}$ is the generation rate of the intrinsic average of k due to the action of the porous matrix.

3. Macroscopic model for moving bed

Here, only cases where the solid phase velocity is kept constant will be considered. The configuration analyzed can be better visualized with the help of the Representative elementary control-volume of Fig. 1. A moving bed crosses a fixed control volume in addition to a flowing fluid, which is not necessarily moving with a velocity aligned with the solid phase velocity. The steps below show first some basic definitions prior to presenting a proposal for a set of transport equations for analyzing the system of Fig. 1.

4. Basic definitions and hypotheses

The first step here is to defined velocities and their averages related to a fixed representative elementary control-volume.

A general form for a volume-average of any property φ, distributed within a phase γ that occupy volume ΔV_γ, can be written as [10],

$$\langle\varphi\rangle^\gamma=\frac{1}{\Delta V_\gamma}\int_{\Delta V_\gamma}\varphi\, dV_\gamma \tag{9}$$

In the general case, the volume ratio occupied by phase γ will be $\phi^\gamma=\Delta V_\gamma/\Delta V$.

If there are two phases, a solid ($\gamma=s$) and a fluid phase ($\gamma=f$), volume average can be established on both regions. Also,

$$\phi^s=\Delta V_s/\Delta V=1-\Delta V_f/\Delta V=1-\phi^f \tag{10}$$

and for simplicity of notation one can drop the superscript "f" to get $\phi^s=1-\phi$

As such, calling the instantaneous local velocities for the solid and fluid phases, $\mathbf{u_s}$ and $\mathbf{u}$, respectively, one can obtain the average for the solid velocity, within the solid phase, as follows,

$$\langle\mathbf{u}\rangle^s=\frac{1}{\Delta V_s}\int_{\Delta V_s}\mathbf{u}_s\, dV_s \tag{11}$$

which, in turn, can be related to an average velocity referent to the entire REV as,

$$\mathbf{u}_S=\frac{\overbrace{\Delta V_s}^{(1-\phi)}}{\Delta V}\underbrace{\frac{1}{\Delta V_s}\int_{\Delta V_s}\mathbf{u}_s\, dV_s}_{\langle\mathbf{u}\rangle^s} \tag{12}$$

A further approximation herein is that the porous bed is rigid and moves with a steady average velocity $\mathbf{u}_S$. Note that the condition of steadiness for the solid phase gives $\mathbf{u}_S=\bar{\mathbf{u}}_S=$const where the overbar denotes, as usual in the literature, time-averaging.

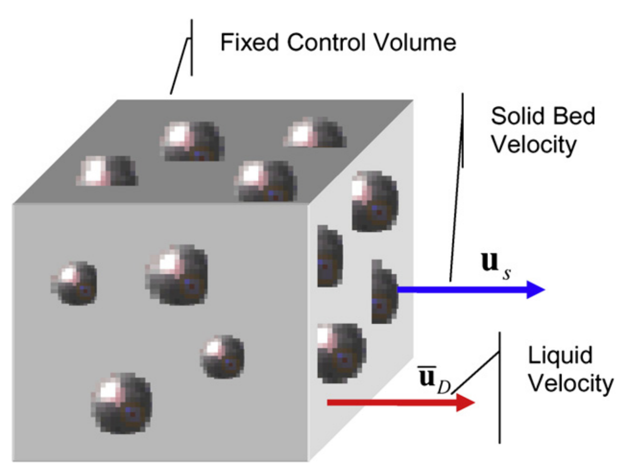

Fig. 1. Representative elementary control-volume for a moving porous bed (REV).

For the fluid phase, the intrinsic (fluid) volume average gives, after using the subscript "i" also for consistency with the literature,

$$\langle \overline{\mathbf{u}} \rangle^i = \frac{1}{\Delta V_f} \int_{\Delta V_f} \overline{\mathbf{u}} \, dV_f \tag{13}$$

Both velocities can then be written as,

$$\overline{\mathbf{u}}_D = \phi \langle \overline{\mathbf{u}} \rangle^i, \mathbf{u}_S = (1-\phi)\langle \mathbf{u} \rangle^s = \text{const.} \tag{14}$$

In the general case, $\overline{\mathbf{u}}_D$ and $\mathbf{u}_S$ need not to be aligned with each other as in the drawing of Fig. 1. For a general three-dimensional flow they are written as,

$$\overline{\mathbf{u}}_D = \overline{\mathbf{u}}_D \hat{i} + \overline{v}_D \hat{j} + \overline{w}_D \hat{k}; \mathbf{u}_S = u_S \hat{i} + v_S \hat{j} + w_S \hat{k} \tag{15}$$

where u,v,w are the Cartesian components.

A total-volume based relative velocity is defined as,

$$\overline{\mathbf{u}}_{\text{rel}} = \overline{\mathbf{u}}_D - \mathbf{u}_S \tag{16}$$

Further,

$$\overline{\mathbf{u}}_{\text{rel}} = \phi \langle \overline{\mathbf{u}} \rangle^i - (1-\phi)\langle \mathbf{u} \rangle^s; \overline{\mathbf{u}}_{\text{rel}} = \phi\left(\langle \overline{\mathbf{u}} \rangle^i + \langle \mathbf{u} \rangle^s\right) - \langle \mathbf{u} \rangle^s \tag{17}$$

The modulus of $\mathbf{u}_{\text{rel}}$ can be calculated as,

$$|\overline{\mathbf{u}}_{\text{rel}}| = |\overline{\mathbf{u}}_D - \mathbf{u}_S| = \sqrt{(\overline{u}_D - u_S)^2 + (\overline{v}_D - v_S)^2 + (\overline{w}_D - w_S)^2} \tag{18}$$

One could also define a phase-volume based relative velocity as,

$$\overline{\mathbf{u}}^{\gamma}_{\text{rel}} = \langle \overline{\mathbf{u}} \rangle^i - \langle \mathbf{u} \rangle^s \tag{19}$$

and the relationship between these two relative velocities becomes,

$$\frac{\overline{\mathbf{u}}_{\text{rel}}}{\overline{\mathbf{u}}^{\gamma}_{\text{rel}}} = \left(\frac{\phi - (1-\phi)\frac{\langle \mathbf{u} \rangle^s}{\langle \overline{\mathbf{u}} \rangle^i}}{1 - \frac{\langle \mathbf{u} \rangle^s}{\langle \overline{\mathbf{u}} \rangle^i}} \right) \tag{20}$$

Although it is recognized that the drag between phases can be related to $\overline{\mathbf{u}}^{\gamma}_{\text{rel}}$, in the equations to follow, for simplicity, $\overline{\mathbf{u}}_{\text{rel}}$ will be used for characterizing the relative movement between phases. Further, for $\langle \mathbf{u} \rangle^s = 0$ the result $\overline{\mathbf{u}}_{\text{rel}} = \phi \overline{\mathbf{u}}^{\gamma}_{\text{rel}}$ is equivalent to $\overline{\mathbf{u}}_D = \phi \langle \overline{\mathbf{u}} \rangle^i$ and for $\langle \mathbf{u} \rangle^s / \langle \overline{\mathbf{u}} \rangle^i = 1$ one gets $\overline{\mathbf{u}}^{\gamma}_{\text{rel}} / \overline{\mathbf{u}}_{\text{rel}} = 0$.

5. Transport equations

Incorporating now in Eq. (6) a model for the Macroscopic Reynolds Stresses $-\rho\phi\langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i$ (see [7–9] for details), and assuming that a relative movement between the two phases is described by Eq. (16), the momentum equation reads,

$$\rho\left[\nabla \cdot \left(\frac{\overline{\mathbf{u}}_D \overline{\mathbf{u}}_D}{\phi}\right)\right] - \nabla \cdot \left\{ \left(\mu + \mu_{t_\phi}\right) \left[\nabla \overline{\mathbf{u}}_D + (\nabla \overline{\mathbf{u}}_D)^T\right] \right\} = -\nabla\left(\phi \langle \overline{p} \rangle^i\right) - \underbrace{\left[\frac{\mu\phi}{K}\overline{\mathbf{u}}_{\text{rel}} + \frac{c_F \phi \rho |\overline{\mathbf{u}}_{\text{rel}}| \overline{\mathbf{u}}_{\text{rel}}}{\sqrt{K}}\right]}_{\text{Viscous and Form drags due to } \overline{\mathbf{u}}_{\text{rel}}} \tag{21}$$

The last two terms in the above equation represent the drag caused by the relative movement between phases. When the two materials flow along with the same velocity, then the fluid feels no extra forces caused by the porous matrix. Pressure head necessary to drive the flow is therefore less then that required to push the fluid through the porous substrate.

A corresponding transport equation for $\langle k \rangle^i$ can be written as,

$$\rho\left[\nabla \cdot \left(\overline{\mathbf{u}}_D \langle k \rangle^i\right)\right] = \nabla \cdot \left[\left(\mu + \frac{\mu_{t_\phi}}{\sigma_k}\right) \nabla\left(\phi \langle k \rangle^i\right)\right] - \rho \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i : \nabla \overline{\mathbf{u}}_D + \underbrace{c_k \rho \frac{\phi \langle k \rangle^i |\overline{\mathbf{u}}_{\text{rel}}|}{\sqrt{K}}}_{\text{Generation rate due to } \overline{\mathbf{u}}_{\text{rel}}} - \rho\phi\langle \varepsilon \rangle^i \tag{22}$$

where the generation rate due to the porous substrate, G^i, which was included in Eq. (7), now depends on $|\overline{\mathbf{u}}_{\text{rel}}|$. If there is now shear between the two phases ($\overline{\mathbf{u}}_{\text{rel}} = 0$), than no mean kinetic energy is additionally transformed into turbulence by the action of porous substrate. In this case, G^i will be of null value. In addition, for a uniform one-dimensional flow, P^i in Eq. (7) will vanish. Therefore, if within such one-dimensional uniform flow the fluid is moving with the same speed as the solid matrix, an initial level of $\langle k \rangle^i$ will die out as there is no existing mechanism to produce turbulent kinetic energy and maintain the initial values for $\langle k \rangle^i$.

6. Application to a moving bed

A numerical example of the above is shown next. The flow under consideration is schematically presented in Fig. 2, where a channel is completely filled with a moving layer of a porous material. The channel shown in the figure has length and height given by L and H, respectively. A constant property fluid flows longitudinally from left to right permeating through the porous structure. Results at the channel center ($y = H/2$) are a representative of uniform one-dimensional fully developed flow after a certain developing length.

7. Numerical details

The above transport equations are discretized in a generalized coordinate system using the control volume method [12]. Faces of the volumes are formed by lines of constant coordinates $\eta - \xi$. All computations were carried out until normalized residues of the algebraic equations were brought down to 10^{-7}. Details of the discretization of all terms in the equations can be found in [8,9].

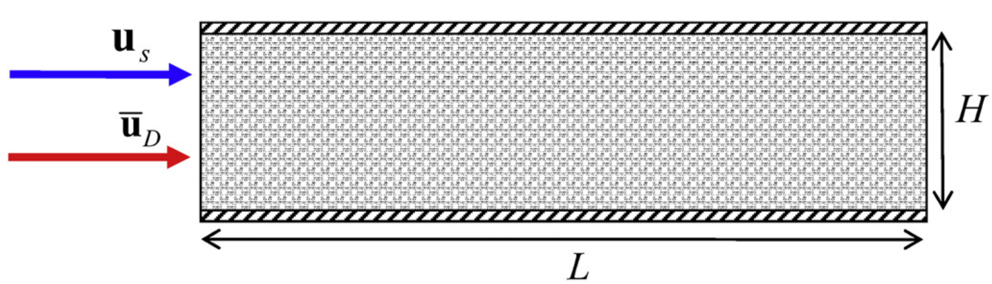

Fig. 2. Porous bed reactor with a moving solid matrix.

Further, the height of the channel section was taken as H=0.075 m and the length L was 0.75 m. For all runs here studied, a total of 200 nodes in the axial direction were used, in addition to 100 nodes in the transversal direction. The results were also checked for grid independence.

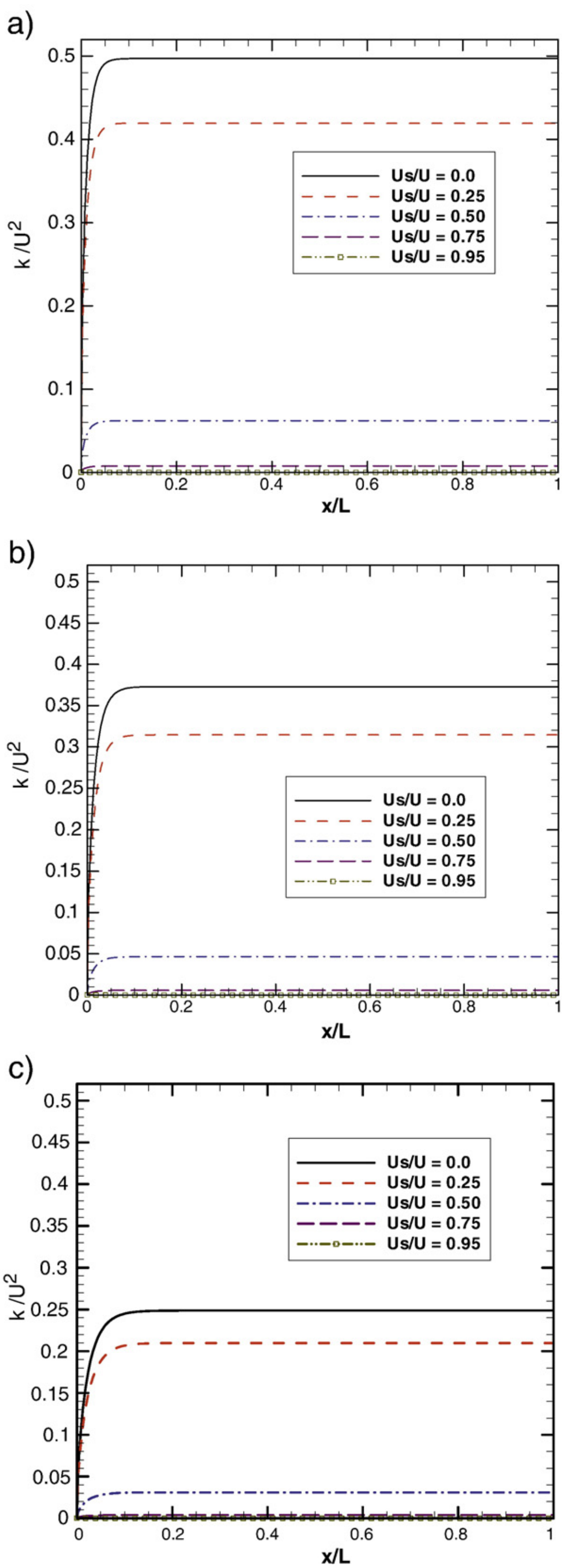

Fig. 3. Effect of $\mathbf{u}_S/\overline{\mathbf{u}}_D$ on non-dimensional turbulent kinetic energy $\langle k\rangle^v/|\overline{\mathbf{u}}_D|^2$: a) ϕ=0.6, b) ϕ=0.7, c) ϕ=0.8.

8. Results and discussion

The flow in Fig. 2 was computed with the set of Eqs. (1), (18), (21), and (22) including constitutive Eq. (3) and the Kolmogorov–Prandtl expression (5). The wall function approach was used for treating the flow close to the walls.

Fig. 3 shows values for of the non-dimensional turbulent kinetic energy $\langle k\rangle^v/|\overline{\mathbf{u}}_D|^2$ along the channel mid-height. In all cases, inlet values for $\langle k\rangle^v/|\overline{\mathbf{u}}_D|^2$ were equal to 5.23×10^{-4}. The figure indicates the damping of turbulence as the solid velocity approaches the fluid velocity. As the relative velocity decreases, the amount of disturbance past the solid obstacles is reduced, implying then in a reduction of the final level of $\langle k\rangle^i$ according to G^i in Eq. (7). For a fixed incoming mass flow rate into the channel (fixed $\overline{\mathbf{u}}_D$), the figure further indicates the effect of porosity ϕ. Low porosities increase the intrinsic fluid velocity $\langle\overline{\mathbf{u}}\rangle^i$, reflecting a greater conversion of mechanical kinetic energy into turbulence [13].

Ultimately, results in Fig. 3 indicate that for flows where the porous bed also moves, in the same direction of the flow, a smaller portion of the available mean mechanical energy is converted into turbulence by the action of the porous substrate. If that is the case occurring in industrial processes, results herein might be useful in analyzing engineering equipment and contributing towards more realistic simulation of flow in moving porous beds.

9. Conclusions

Numerical solutions for turbulent flow in a moving porous bed were obtained for different ratios $\mathbf{u}_S/\overline{\mathbf{u}}_D$. Governing equations were discretized and numerically solved. Increasing the solid speed reduces the interfacial drag, which minimizes the conversion of mean mechanical energy into turbulence. Results herein may contribute to the design and analysis of engineering equipment where a moving porous body is identified.

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