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ABSTRACT

The present analysis accounts for combined convective and radiant heat transfer to a fluid flowing in a short tube with prescribed wall heat flux. The heat flux distribution used was of sine shape with maximum at the middle of the tube. This solution is known to represent the axial power variation in a nuclear reactor core. The tube wall and gas bulk temperatures were obtained by successive substitutions for the wall and gas energy balance equations. The integrals were approximated by Sympson's rule and initial guesses for the iterative process were based upon limiting cases for pure radiation and pure convection. The results of the combined solution compared with the pure radiation approach show a decrease of 30 percent for the maximum wall temperature using black surface (ε=1). For this same situation, the increase in the gas temperature along the tube shows a reduction of 58 percent when compared to the pure convection solution.

Introduction

High temperature heat transfer devices are often associated with large heat fluxes which may occur by radiation and convection mechanisms. The superimposed effect of radiation is found mainly in cooling hot spots by transporting heat to colder regions. In some instances, this transport will impose an additional load on a place which is to be kept cool, and hence, this amount of energy must be estimated. In other cases, radiation will help to cool an undesirably heated region.

In the case of a nuclear reactor core, either rod-bundles or axial holes through the core material, axial transport of heat might be important in predicting high temperature locations in steady state and abnormal conditions. In those devices, the axial power can be approximated by a sinusoidal function of the axial position.

Based on the foregoing application, studies were done on the effect of coupled radiation and convection in a circular channel with a specified sinusoidal wall heat flux and flow of a transparent gas. The convective heat transfer coefficient h, was assumed constant throughout the tube. As pointed out by Kays[1] for the case of a gas flowing in a channel ($Pr \sim 1$), the varying heat flux has little influence on h, and it is perfectly adequate to use a Nusselt number based upon constant heat flux theory.

In solving the integral energy equation for the heat balance at the wall, the integrals were numerically calculated by making use of limiting case solutions for pure radiation and pure convection as initial guesses. Subsequent iterations using previous output were performed until convergence was obtained within some pre-selected tolerance. Such an approach will be explained later, and has been used successfully to solve a wide variety of problems, as in Sparrow et al [2], and Sparrow and Jonsson[3].

Another common procedure to solve radiative integral equations is the Separable Kernal Method (SKM), which consists basically in the transformation of the integral equation into a differential equation by means of approximating the radiation Kernel as a separable function.

Literature Review

Pure radiation[4], as well as combined radiation and convection in tubes[5,6], have been extensively studied in the literature.

Usiskin and Siegel[4] obtained a numerical solution for radiation transport from short tubes by dividing the tube length into several isothermal sections. A heat balance on each of these regions was taken, resulting in a set of nonlinear algebraic equations which was solved for the wall temperature in each isothermal zone.

Perlmutter and Siegel[5] presented an analysis of a problem similar to the present one, but a constant wall heat flux was studied. In Ref. 5, numerical solutions for a black surface case were carried out to show the effect of each of the independent parameters, such as the gas inlet and environment temperatures. Two methods were employed: the SKM and direct numerical solution of the integral equation. The first led to the transformation of the energy balance into a second-order differential equation which, together with a first-order differential equation for the gas temperature, were integrated numerically. In the second method, the integral equation was put into finite-difference form and the set of nonlinear algebraic relations obtained was solved by the Newton-Raphson method. This second solution was performed with the aim of checking the first one. Results were good for short tubes and calculations for long tubes were not done due to the number of algebraic equations involved. Siegel and Perlmutter[6] used the same geometry as in Ref. 5 and extended their analysis to a gray surface case.

The successive substitution scheme for solving linear integral equations can be carried out by a number of methods and is well documented in the literature. Sparrow[2,3] obtained solution for cavity problems where the equations were linear. In Ref. 7 it is pointed out that nonlinear problems concerning coupling of conduction and convection are still able to be solved by such method if appropriate modifications are made.

Analysis

The system to be analyzed in the present work is shown schematically in Fig. 1. The surface of a circular tube with diameter D and length L has been superimposed by an axisymmetric heat flux $q_w(X)$, with sinusoidal shape and maximum heat flux at $X = L/2$. The gas enters the tube with bulk temperature T_{gi} and mean velocity u_m, and leaves with bulk temperature T_{ge}. The ends of the tube are exposed to a medium or environment which will exchange heat with the inside tube surface elements. At the inlet, this medium is at the arbitrary temperature T_{ri}, and at the outside the temperature is T_{re}. The inside surface is assumed to be a gray diffused one, with emissivity ε independent of temperature level. In addition, considering gas flow ($Pr \cong 0.7$) and temperature variations expected, fluid properties can be kept constant.

Energy Balance

The energy balance will be derived following the Radiosity method, also used in Ref. 6. In this method, the total radiant flux leaving an element of surface is the difference between the outgoing radiant flux q_o and the incoming radiant flux q_i. The outgoing radiant flux q_o is composed by a part emitted by the element itself, and a part reflected of the incoming radiant flux, which in turn represents the leaving flux from another element. Therefore, a balance of the net flux leaving the surface by radiation, convection, and supplied wall heat flux can be stated as follows

$$q_w(X) + q_i = q_o + h[T_w(X) - T_g(X)] \ . \tag{1}$$

In Eq. 1, the last term on the right represents the local convective heat flux, which is proportional to the local temperature difference $[T_w(X) - T_g(X)]$, since the film coefficient h is kept constant.[1]

The radiative flux leaving a surface element can be written as

$$q_o = \varepsilon\sigma T_w^4 + (1 - \varepsilon)q_i \ . \tag{2}$$

The incoming radiant flux is composed by radiation coming from flux leaving other surface elements and radiation coming from the environment through the tube ends. So we have

$$q_i = \sigma T_{ri}^4 F(x) + \sigma T_{re}^4 F(\lambda - x)$$

$$+ \int_0^x q_o(\xi)K(x - \xi)d\xi + \int_x^\lambda q_o(\xi)K(\xi - x)d\xi \ , \tag{3}$$

where $x = X/D$ and $\lambda = L/D$.

The geometric configuration factor F(x) from an element of surface to the tube end is given by Ref. 7 as follows:

$$F(x) = [(x^2 + 1/2)/(x^2 + 1)^{1/2}] - x \qquad x \geq 0 \ , \tag{4}$$

and the configuration factor K(x) between two rings inside the tube is[6]

[1]As pointed out in Ref. 6, a variation on h could have been used in this approach. Nevertheless, as already commented, such a procedure is worthless regarding the accuracy obtained since we are dealing with a gas having a Prandtl number of the order of 1[1].

$$K(x) = 1 - \left[\left(x^3 + 3x/2\right)/\left(x^2 + 1\right)^{3/4}\right] \qquad x \geq 0 \; . \tag{5}$$

Substituting Eqs. 2 and 3 into Eq. 1, we get

$$q_w(x) + \sigma T_{ri}^4 \; F(x) + \sigma T_{re}^4 \; F(\lambda - x)$$

$$+ \int_0^x q_o(\xi)K(x - \xi)d\xi \; + \int_x^\lambda q_o(\xi)K(\xi - x)d\xi$$

$$= h\left[T_w(x) - T_g(x)\right] + q_o(x) \; . \tag{6}$$

Using Eqs. 1 and 2, we can write

$$q_o = \varepsilon\sigma T_w^4 + (1 - \varepsilon)\left[q_o + h\left(T_w - T_g\right) - q_w(x)\right] \; ,$$

or

$$q_o = \left[(1 - \varepsilon)/\varepsilon\right]\left[h\left(T_w - T_g\right) - q_w(x)\right] + \sigma T_w^4 \; . \tag{7}$$

To introduce another equation to be solved together with Eq. 6, a balance of energy in a cylindrical element of fluid of length dx is required. The rate of increase of internal energy for the fluid crossing dx can be written as

$$\rho u_m c_p \left(\pi D^2/4\right) dT \; .$$

Since the fluid is considered transparent, this quantity should be equal to the convective heat flux only. Therefore, we have

$$h\pi D dx\left[T_w(x) - T_g(x)\right] = \rho u_m c_p\left(\pi D^2/4\right) dT \; ,$$

or

$$dT/dx = S\left[T_w(x) - T_g(x)\right] \; , \tag{8}$$

where

$$S = 4\left(\mu/\rho u_m D\right)\left(\kappa/\mu c_p\right)(hD/\kappa)$$

is the Stanton number.

Integrating Eq. 8 from 0 to x,

$$T = T_{gi} + \int_0^x S[T_w(x) - T_g(x)]\, dx \,. \qquad (9)$$

The energy balances for the wall surface and gas yield a system of two equations and two variables, $T_w(x)$ and $T_g(x)$ that can be solved numerically.

The specified wall heat flux $q_w(x)$, will now be introduced since Eq. 6 is written for any distribution of $q_w(x)$.

The function considered is

$$q_w(x) = q^1 \sin(\pi x/\lambda) + q^2 \,.$$

This heat flux is the approximate solution for the power generated in a nuclear reactor core. Finally, writing equations for $T_w(x)$, $T_g(x)$, and $q_w(x)$ introducing nondimensional parameters, defined as

$$H = (h/q_*)(q_*/\sigma)^{1/4} \;; \qquad t = (\sigma/q_*)^{1/4}\, T \;;$$

$$q_*^1 = q^1/q_* \;; \qquad q_*^2 = q^2/q_* \;;$$

$$q_w^+ = q_w/q_*; \qquad q_o^+ = q_o/q_*$$

where

q_* = reference flux for nondimensionality,

we get

$$q_w^+(x) + t_{ri}^4\, F(x) + t_{re}^4\, F(\lambda - x)$$

$$+ \int_0^x q_o^+(\xi)K(x-\xi)d\xi + \int_x^\lambda q_o^+(\xi)K(\xi - x)d\xi$$

$$= h[t_w(x) - t_g(x)] + q_o^+(x) \qquad (10)$$

$$t_g(x) = t_{gi} + \int_o^x S[t_w(x) - t_g(x)]\ dx \tag{11}$$

where

$$q_w^* = q_*^1 \sin(\pi x/\lambda) + q_*^2 \tag{12}$$

$$q_o^* = \frac{(1-\varepsilon)}{\varepsilon}\{h[t_w(x) - t_g(x)] - q_w^+\} + t_w^4(x) \ . \tag{13}$$

Equations 10 and 11 were solved numerically for $q_*^1 = 0$ and $q_*^2 = 0$. The limiting cases, numerical procedure, and results are presented below.

Limiting Cases

Pure Radiation Solution

In this solution, it will be assumed the exponential kernel approximation (SKM) which is known to give good results for short tubes[6].

Not considering the convection term in Eq. 6, it can be written as

$$q_w(x) + \sigma T_{ri}^4\ F(x) + \sigma T_{re}^4\ F(\lambda - x)$$

$$+ \int_o^x q_o(\xi)K(x-\xi)d\xi + \int_x^\lambda q_o(\xi)K(\xi - x)d\xi = q_o(x) \ . \tag{14}$$

Since the above equation is linear in q_o, we may divide the problem as shown in Fig. 2. The final solution will be the sum of the three cases.

Solution for I

Equation 14, using the SKM approximation, is

$$q_o(x) = q_w(x) + \frac{1}{e^{2x}}\int_o^x q_o(\xi)\ e^{2\xi}\ d\xi + \int_x^\lambda q_o(\xi)\ \frac{1}{e^{2\xi}}\ d\xi \ . \tag{15}$$

Differentiating it twice, and subtracting from Eq. 15,

$$d^2 q_o/dx = -\ q^1[(\pi/\lambda)^2 + 4]\ \sin(\pi x/\lambda) - 4q^2 \ .$$

Integrating twice,

$$q_o = q^1[1 + (2\lambda/\pi)^2] - 2q^2x^2 + C_1x + C_2 .$$

By symmetry, $dq_o/dx = 0$ at $x = \lambda/2$, so the constant C_1 can be determined as $2q^2\lambda$. To determine C_2, we evaluate Eq. 14 at points $x = 0$ and $x = \lambda$, and use the symmetry relation $T_w(0) = T_g(\lambda)$. After doing that, C_2 is found to be $q^2(\lambda + 1)$.

Then the solution for I is

$$q_o = q^1[1 + (2\lambda/\pi)^2] \sin(\pi x/\lambda) + q^2[\lambda + 1 + 2(x\lambda - x^2)]. \tag{16}$$

Solution for II

Due to equilibrium assumed for the problem, the solution for this case is

$$q_o = \sigma T_w^4(x) = \sigma T_{re}^4 . \tag{17}$$

Solution for III

The balance equation for this case is

$$q_o(x) = \sigma\left(T_{ri}^4 - T_{re}^4\right) F(x) + \int_0^x q_o(\xi)K(x - \xi)d\xi + \int_x^\lambda q_o(\xi)K(\xi - x)d\xi \tag{18}$$

Again, differentiating twice and subtracting from Eq. 18, we find $d^2q_o/dx = 0$, so $q_o = C_3x + C_4$. As before, the constants can be found by applying symmetry at the middle and ends of the tube. The solution is then

$$q_o = (.5 + \lambda - x)\, \sigma\left(T_{ri}^4 - T_{re}^4\right)/(1 + \lambda) \tag{19}$$

The final solution is the sum of Eqs. 16, 17, and 19, and after using nondimensional parameters, we get

$$q_o^+(x) = q_*^1 \left[1 + \left(\frac{2\lambda}{\pi}\right)^2\right] \sin\frac{\pi x}{\lambda} + q_*^2[\lambda + 1 + 2(x\lambda - x^2)]$$

$$+ t_{re}^4 \left(\frac{t_{ri}^4 - t_{re}^4}{1 + \lambda} \right) (0.5 + \lambda - x) \ , \tag{20}$$

and

$$t_w^4 = \left(\frac{1 - \varepsilon}{\varepsilon} \right) q_w^+ + q_o^+ \qquad \text{from Eq. 7.}$$

For the case of a constant heat input, $q_*^1 = 0$. For sinusoidal flux vanishing at ends, $q_*^2 = 0$.

Pure Convection Solution

The pure convection solution is straightforward and will just be presented here. For a wall heat flux described by Eq. 10, using constant h, we get, after integrating the balance equation, and nondimensionalizing

$$t_g(x) - t_{gi} = \frac{\lambda S}{\pi H} q_*^1 [1 - \cos(\pi x/\lambda)] + \frac{S}{H} q_*^2 x \tag{21}$$

and

$$t_w(x) - t_{gi} = \frac{q_*^1}{H} \left\{ \frac{S\lambda}{H} \left[1 - \cos\left(\frac{\pi x}{\lambda}\right) \right] + \sin\left(\frac{\pi x}{\lambda}\right) \right\}$$

$$+ \frac{q_*^1}{H} (Sx + 1) \tag{22}$$

Numerical Procedure

The solution of Eqs. 10 and 11 was obtained by first guessing the profiles for t_g and t_w. The integrals were then calculated in order to obtain an improved estimate. Subsequent iterations were performed using the most recent results until no significant variation in the variables could be detected.

For the wall temperature, the first guess was the pure radiation limiting case, since it was known to be closer to the actual solution[6]. For the gas temperature, an initial profile based on the pure convection solution gave faster convergence.

The integrals in Eq. 10 were performed using Sympson's rule. A total of 50 axial segments were used for a tube length of 5 diameters. The convergence criterion adopted consisted of comparing the integral of the difference between the last two trials with some pre-selected error. This error was in the range of 0.0001 for the present calculations.

For high emissivity cases, an under-relaxation factor of 0.9 was used to multiply the recently obtained solution. Therefore, the previous solution was weighted in 10 percent. For low emissivities, the rate of convergence was even more decelerated. In this case, the relaxation factor had to be as low as 0.7 to avoid oscillations in the calculated profiles.

Results and Discussion

For the sake of simplicity and ease of comparison in all the computations here presented, the gas and environment were assigned the same temperature at both inlet and exit sides. This procedure reduced the number of independent variables. In addition, the nondimensional inlet gas temperature in all cases was taken as 1.5.

In order to check the accuracy of the program, the limiting case of pure radiation was simulated setting adequate parameters. Figure 3 shows results obtained for constant wall heat flux when the Stanton number S, and the nondimensional film coefficient H, were set to zero. Superposition of the two solutions indicates the correctness of the programming, regardless of the emissivity used.

Figure 4 presents results for S = 0.01, H = 0.8, and constant wall heat flux. The values for S and H were calculated from the fully turbulent correlation $Nu = 0.023\ Re^{0.8} * Pr^{0.4}$ for Re = 100000 and Pr = 0.7. Figure 5 presents calculations using the same input parameters as above but sinusoidal wall heat flux is used. For both fluxes, the figures show a substantial decrease for the maximum wall temperature when surface elements are allowed to exchange heat by radiation with each other and with the environment. As expected, this exchange is more effective when the surface emissivity has its maximum value. For the sinusoidal flux, Fig. 5 shows a wall temperature decrease of 30 percent. In both cases, the combined solution approximates the pure convection solution as the ability of a surface to emit radiant energy decreases. This tendency was already observed in Ref. 6.

The increase in the gas temperature presents a decrease of 58 percent for black body solution and sine flux. The results show that the higher the emissivity, the smaller the amount of heat transferred to the gas, and since the wall temperatures are reduced, more heat is transferred directly to the open ends. Such behavior would not be expected for a long tube where the convective mechanism certainly plays the most important role[6]. However, in a nuclear reactor, the number of voids through the core may contribute in some extent for direct exchange of heat with both upper and lower plena.

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Nomenclature

c_p = Specific heat of fluid
D = Tube diameter
F = Geometric configuration factor for radiation from an element on the wall to the circular opening at the end of the tube
H = Dimensionless heat transfer coefficient $(h/q_*)(q_*/\sigma)^{1/4}$
h = Convective heat transfer coefficient
K = Geometric configuration factor between elements on inside of tube wall
k = Thermal conductivity of gas
L = Length of tube
λ = Dimensionless length, L/D
Nu = Nusselt number hD/k
Pr = Prandtl number $c_p\mu/k$
q = Heat per unit of area at tube wall
Re = Reynolds number $u_m D\rho/\mu$
S = Stanton number 4 Nu/Re Pr = $4h/u_m\rho c_p$
T = Temperature
t = Dimensionless temperature $(\sigma/q_*)^{1/4}t$
u_m = Mean gas velocity
X = Axial length coordinate measured from tube entrance
x = Dimensionless coordinate X/D
ε = Emissivity of surface
μ = Viscosity of gas
ξ = Dimensionless integration variable
ρ = Density of gas
σ = Stefan-Boltzmann constant

Subscripts

e = Exit end of tube

g = Gas

i = Inlet end of tube, incoming

o = Outgoing

r = Reservoir

* = Nondimensional heat flux

w = wall

Superscripts

1 = Referent to sin term in wall heat input equation

2 = Referent to const. term in wall heat input equation

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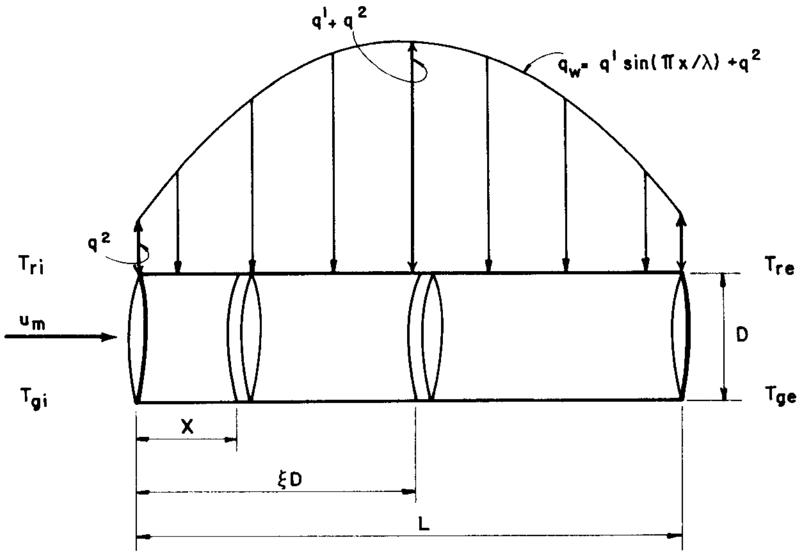

FIG. 1 - CIRCULAR TUBE GEOMETRY

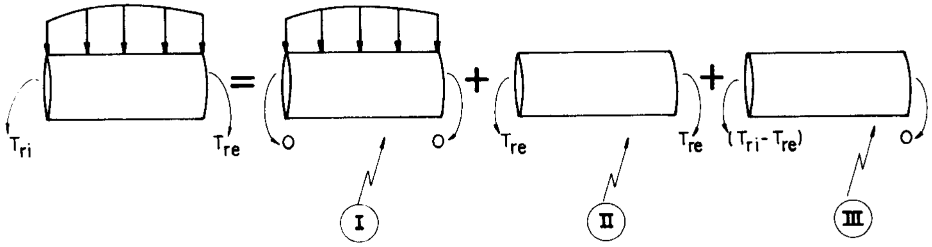

FIG. 2 - PURE RADIATION SOLUTION

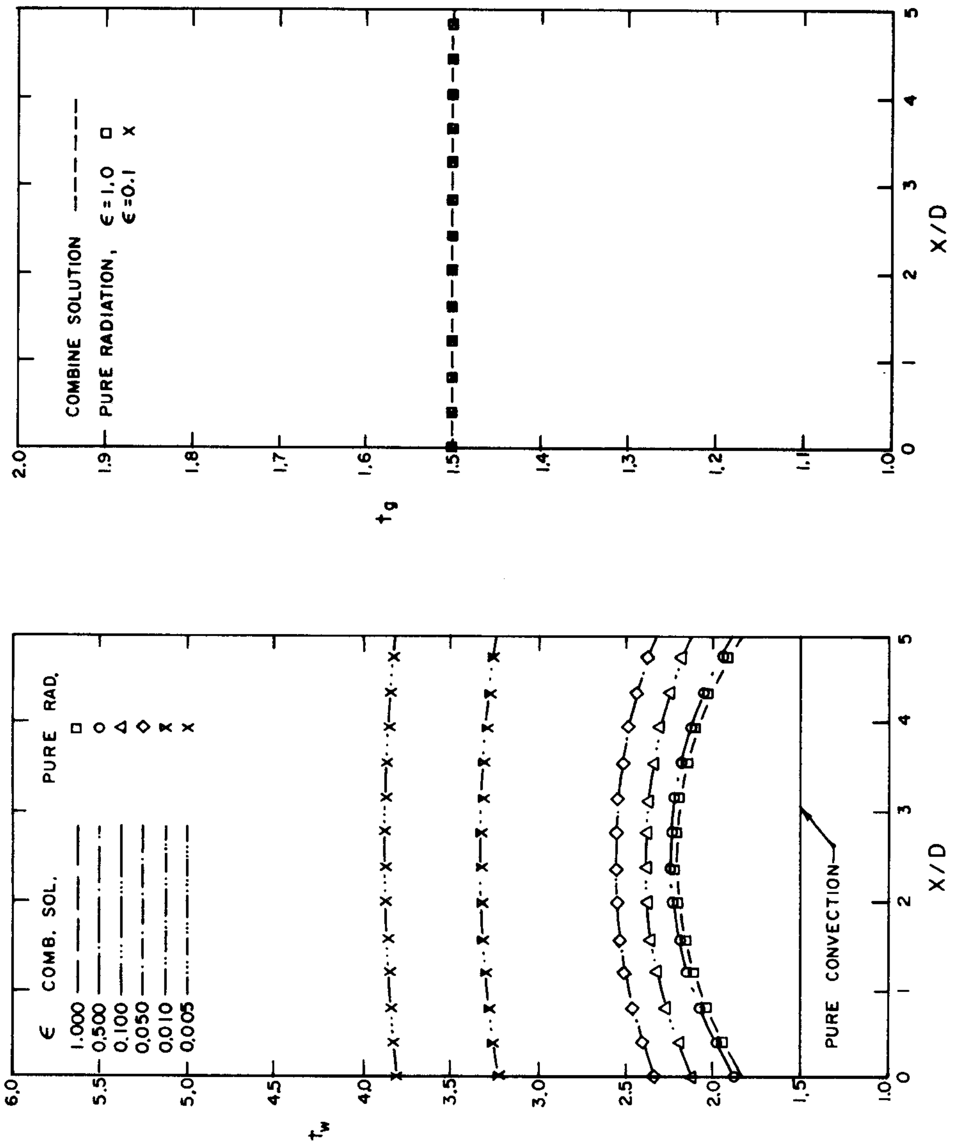

FIG. 3 Nondimensional Wall and Gas Temperatures. Pure Radiation Simulation With Constant Wall Heat Flux, S = 0, H = 0, t_{gi} = 1.5, q_*^1 = 0, and q_*^2 = 1.

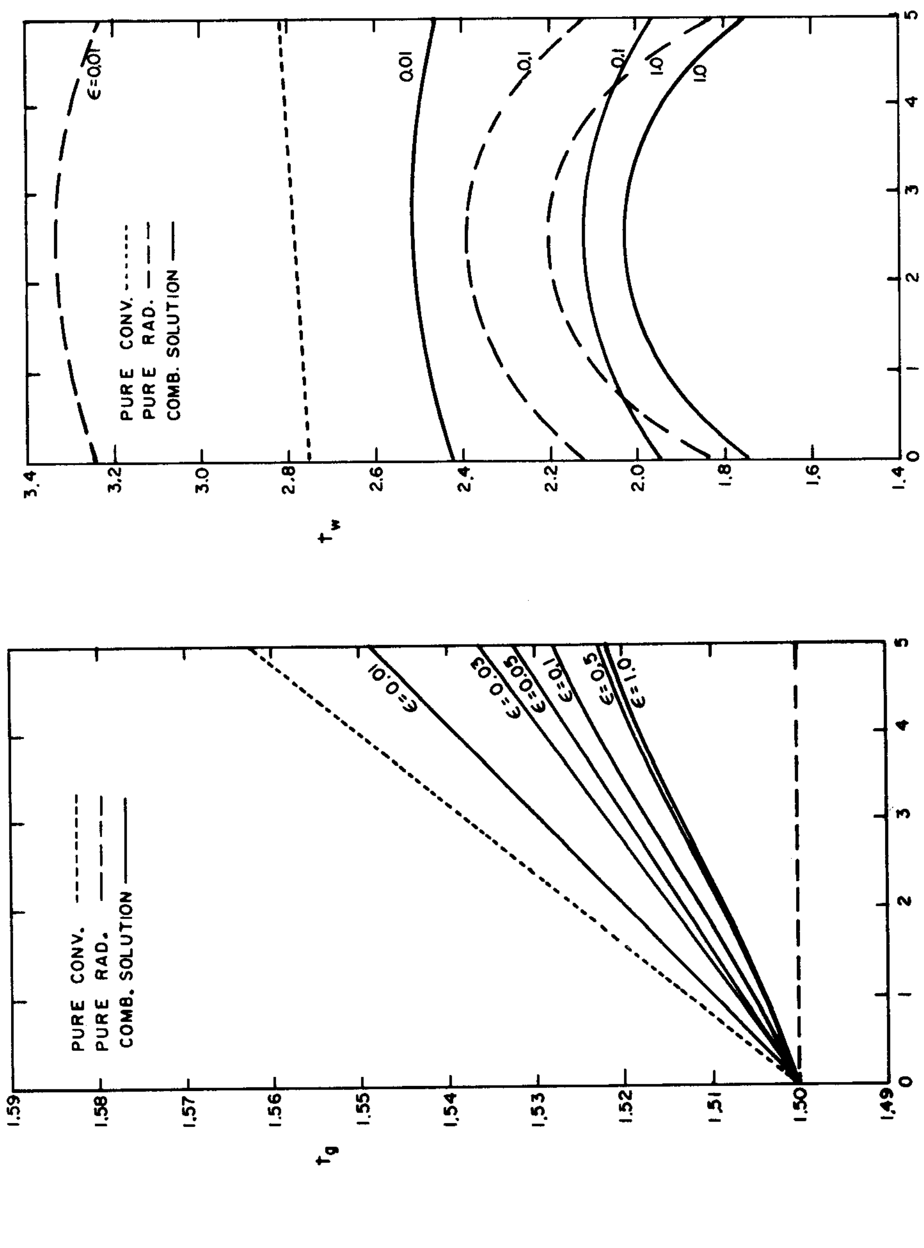

FIG. 4 Nondimensional Wall and Gas Temperatures. Constant Wall Heat Flux, $S = 0.01$, $H = 0.8$, $t_{gi} = 1.5$, $q_*^1 = 0$, and $q_*^2 = 1$.