Pergamon

Int. Comm. Heat Mass Transfer, Vol. 30, No. 5, pp. 615-624, 2003

0735–1933/03/$–see front matter

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PII: S0735-1933(03)00099-X

MODELING OF TURBULENT NATURAL CONVECTION IN POROUS MEDIA

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(Communicated by J.P. Hartnett and W.J. Minkowycz)

ABSTRACT

This paper applies the volume-average mathematical operator on the buoyancy term in the flow equations governing turbulent flow. Volume averaging is taken on both mean and turbulent fields. Derivations are carried out under the recently established double-decomposition concept. Results show that additional buoyancy generation term appears if both time and volume averaging procedures are simultaneously applied. Final modeled equations are based on a macroscopic k-ε model for porous media.

Introduction

This paper presents an analysis of macroscopic buoyancy effects in turbulent flow in porous media. Traditionally, modeling of macroscopic transport for incompressible flows in such media has been based on the volume-average methodology for either heat [1] or mass transfer [2-5]. If time fluctuations of the flow properties are also considered, in addition to spatial deviations, there are two possible methodologies to follow in order to obtain macroscopic equations: *a*) application of time-average operator followed by volume-averaging [6-9], or *b*) use of volume-averaging before time-averaging is applied [10-13]. In fact, these two sets of macroscopic transport equations are equivalent when examined under the recently established *double decomposition* concept [14-17]. This methodology, initially developed for the flow variables, has been extended to non-buoyant heat transfer in porous media where both time fluctuations and spatial deviations were considered for velocity and temperature [18-19]. Recently, a general classification of all proposed models for turbulent flow and heat transfer in porous media has been published [20]. Here, buoyant flows are considered.

Microscopic Instantaneous Equations

The steady-state microscopic instantaneous transport equations for an incompressible fluid with constant properties are given by:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\rho \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} \tag{2}$$

$$(\rho c_p) \nabla \cdot (\mathbf{u} T) = \nabla \cdot (k \nabla T) \tag{3}$$

where $\mathbf{u}$ is the velocity vector, ρ is the density, p is the pressure, μ is the fluid viscosity, $\mathbf{g}$ is the gravity acceleration vector, c_p is the specific heat, T is the temperature and k is the fluid thermal conductivity.

If one considers that the density in the last term of (2) varies with temperature, for natural convection flow, the Boussinesq hypothesis reads, after renaming this density ρ_T,

$$\rho_T \cong \rho[1 - \beta(T - T_{ref})] \tag{4}$$

where the subscript *ref* indicates a reference value and β is the thermal expansion coefficient defined by,

$$\beta = -\frac{1}{\rho} \frac{\partial \rho}{\partial T}\bigg|_p \tag{5}$$

Equation (4) is an approximation of (5) and shows how density varies with temperature in the body force term of the momentum equation.

Further, substituting (4) into (2), one has,

$$\rho \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g}[1 - \beta(T - T_{ref})] \tag{6}$$

Thus, the momentum equation becomes,

$$\rho \nabla \cdot (\mathbf{u}\mathbf{u}) = -(\nabla p)^* + \mu \nabla^2 \mathbf{u} - \rho \mathbf{g} \beta(T - T_{ref}) \tag{7}$$

where $(\nabla p)^* = \nabla p - \rho \mathbf{g}$ is a modified pressure gradient.

The Double Decomposition Concept

The double decomposition idea, herein used for obtaining macroscopic buoyancy equations, has been detailed in references [14,15] so that only a brief overview is here presented. Further, the resulting equations using this concept for the flow [16, 17] and non-buoyant thermal fields [18, 19] are already available in the literature and for that they are not repeated here. The novelty in this work is the treatment of the buoyancy term in (2).

Basically, for porous media analysis, a macroscopic form of the governing equations is obtained by taking the volumetric average of the entire equation set. In that development, the porous medium is considered to be rigid and saturated by an incompressible fluid.

As such, the volume average of a general property φ taken over a *Representative Elementary Volume*, (REV), in a porous medium can be written as [21-23],

$$\langle\varphi\rangle^{v}=\frac{1}{\Delta V}\int_{\Delta V}\varphi\, dV \tag{8}$$

The value $\langle\varphi\rangle^{v}$ is defined for any point **x** surrounded by a REV of size ΔV. This average is related to the *intrinsic* average for the fluid phase as,

$$\langle\varphi_f\rangle^{v}=\phi\langle\varphi_f\rangle^{i} \tag{9}$$

where $\phi=\Delta V_f/\Delta V$ is the medium porosity and ΔV_f is the volume occupied by the fluid in a REV. Furthermore, one can write,

$$\varphi=\langle\varphi\rangle^{i}+{}^{i}\varphi \tag{10}$$

with $\langle{}^{i}\varphi\rangle^{i}=0$. In equation (10), ${}^{i}\varphi$ is the *spatial deviation* of φ with respect to the intrinsic average $\langle\varphi\rangle^{i}$. Also, the local volume average theorem can be expressed as [21-23]:

$$\begin{aligned}\langle\nabla\varphi\rangle^{v}&=\nabla(\phi\langle\varphi\rangle^{i})+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\,\varphi\, dS\\ \langle\nabla\cdot\boldsymbol{\varphi}\rangle^{v}&=\nabla\cdot(\phi\langle\boldsymbol{\varphi}\rangle^{i})+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\boldsymbol{\varphi}\, dS\end{aligned} \tag{11}$$

where **n** is the unit vector normal to the fluid-solid interface and A_i is the interface area within the REV. It is important to emphasize that A_i should not be confused with the surface area surrounding volume ΔV.

Further, the time average of a general quantity φ is defined as,

$$\overline{\varphi}=\frac{1}{\Delta t}\int_{t}^{t+\Delta t}\varphi\, dt \tag{12}$$

where the time interval Δt is small compared to the fluctuations of the average value, $\overline{\varphi}$, but large enough to capture turbulent fluctuations of φ. Time decomposition can then be written as,

$$\varphi=\overline{\varphi}+\varphi' \tag{13}$$

with $\overline{\varphi'}=0$. Here, φ' is the *time fluctuation* of φ around its average value $\overline{\varphi}$.

Pedras and de Lemos [14,15] showed that for a rigid, homogeneous porous medium saturated with an incompressible fluid, the following relationships apply:

$$\overline{\langle\varphi\rangle^{i}}=\langle\overline{\varphi}\rangle^{i};\ {}^{i}\overline{\varphi}=\overline{{}^{i}\varphi};\ \langle\varphi'\rangle^{i}=\langle\varphi\rangle^{i\,\prime} \tag{14}$$

Therefore, a general quantity φ can be expressed by either,

$$\varphi=\overline{\langle\varphi\rangle^{i}}+\langle\varphi\rangle^{i\,\prime}+\overline{{}^{i}\varphi}+{}^{i}\varphi' \quad \text{or} \quad \varphi=\langle\overline{\varphi}\rangle^{i}+{}^{i}\overline{\varphi}+\langle\varphi'\rangle^{i}+{}^{i}\varphi' \tag{15}$$

Expressions (15) comprise the double decomposition concept of [14, 15] with $\langle{}^{i}\varphi'\rangle^{i}=\overline{{}^{i}\varphi'}=0$.

Time Averaged Transport Equations

In order to apply the time average operator to equations (1), (3) and (7), one considers,

$$\mathbf{u}=\overline{\mathbf{u}}+\mathbf{u}'\;;\; T=\overline{T}+T'\;;\; p=\overline{p}+p' \tag{16}$$

Substituting (16) into the governing equations and considering constant property flow,

$$\nabla\cdot\overline{\mathbf{u}}=0 \tag{17}$$

$$\rho\nabla\cdot(\overline{\mathbf{u}}\,\overline{\mathbf{u}})=-(\nabla\,\overline{p})^{*}+\mu\nabla^{2}\overline{\mathbf{u}}+\nabla\cdot(-\rho\overline{\mathbf{u}'\mathbf{u}'})-\rho\mathbf{g}\beta(\overline{T}-T_{ref}) \tag{18}$$

$$(\rho c_p)\nabla\cdot(\overline{\mathbf{u}}\overline{T})=\nabla\cdot(k\nabla\overline{T})+\nabla\cdot(-\rho c_p\overline{\mathbf{u}'T'}) \tag{19}$$

For clear fluid, the use of the eddy-diffusivity concept for expressing the *stress – strain rate* relationship for the Reynolds stress appearing in (18) gives,

$$-\rho\overline{\mathbf{u}'\mathbf{u}'}=\mu_t\,2\overline{\mathbf{D}}-\frac{2}{3}\rho k\,\mathbf{I} \tag{20}$$

where $\overline{\mathbf{D}}=[\nabla\overline{\mathbf{u}}+(\nabla\overline{\mathbf{u}})^{T}]/2$ is the mean deformation tensor, $k=\overline{\mathbf{u}'\cdot\mathbf{u}'}/2$ is the turbulent kinetic energy per unit mass, μ_t is the turbulent viscosity and $\mathbf{I}$ is the unity tensor. Similarly, for the turbulent heat flux on the r.h.s. of (19) the eddy diffusivity concept reads,

$$-\rho c_p\overline{\mathbf{u}'T'}=c_p\frac{\mu_t}{\sigma_T}\nabla T \tag{21}$$

where σ_T is known as the turbulent Prandtl number.

Further, a transport equation for the turbulent kinetic energy is obtained by multiplying first, by $\mathbf{u}'$, the difference between the instantaneous and the time-averaged momentum equations. Thus, applying further the time average operator to the resulting product, one has,

$$\rho\nabla\cdot(\overline{\mathbf{u}}k)=-\rho\nabla\cdot\overline{\left[\mathbf{u}'\left(\frac{p'}{\rho}+q\right)\right]}+\mu\nabla^{2}k+P_k+G_k-\rho\varepsilon \tag{22}$$

where $P_k=-\rho\overline{\mathbf{u}'\mathbf{u}'}:\nabla\overline{\mathbf{u}}$ is the generation rate of k due to gradients of the mean velocity and

$$G_k=-\rho\beta\mathbf{g}\cdot\overline{\mathbf{u}'T'} \tag{23}$$

is the buoyancy generation rate of k. Also, $q=\dfrac{\mathbf{u}'\cdot\mathbf{u}'}{2}$.

Macroscopic Equations for Non-buoyant Flows

As mentioned, macroscopic time-averaged equations for turbulent non-buoyant flow in porous media have been shown in previous works [14-19] and the sole proposition herein is to include the buoyancy term in that analysis. In summary, previous work considered the following property decompositions based on equations (10) and (15),

$$\overline{\mathbf{u}} = \langle \overline{\mathbf{u}} \rangle^i + {}^i\overline{\mathbf{u}}\,; \quad \overline{T} = \langle \overline{T} \rangle^i + {}^i\overline{T}\,; \quad \overline{p} = \langle \overline{p} \rangle^i + {}^i\overline{p}$$
$$\mathbf{u}' = \langle \mathbf{u}' \rangle^i + {}^i\mathbf{u}'\,; \quad T' = \langle T' \rangle^i + {}^iT'\,; \tag{24}$$

Thus, plugging (24) into (17), (18), (19) and (22), followed by application of the operators (8) and (11), macroscopic equations were obtained. Dropping the superscript "*" in the pressure gradient for simplicity and including additional terms to account for the forces exerted on the fluid by the porous matrix, the final macroscopic momentum equation reads [14-20],

$$\rho\nabla\cdot\left(\frac{\overline{\mathbf{u}}_D\overline{\mathbf{u}}_D}{\phi}\right) = -\nabla(\phi\langle\overline{p}\rangle^i) + \mu\nabla^2\overline{\mathbf{u}}_D + \nabla\cdot(-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i) - \left[\frac{\mu\phi}{K}\overline{\mathbf{u}}_D + \frac{c_F\phi\rho|\overline{\mathbf{u}}_D|\,\overline{\mathbf{u}}_D}{\sqrt{K}}\right] \tag{25}$$

where the buoyancy term has been temporarily dropped for the sake of clarity, the Dupuit-Forchheimer relationship, $\overline{\mathbf{u}}_D = \phi\langle\overline{\mathbf{u}}\rangle^i$, has been used, $\langle\overline{\mathbf{u}}\rangle^i$ identifies the intrinsic (liquid) average of the local velocity vector $\overline{\mathbf{u}}$, K is the medium permeability and c_F is the Forchheimer coefficient. Further, the **Macroscopic Reynolds Stress Tensor** (MRST) is given in reference [15] based on equation (20) as,

$$-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i = \mu_{t_\phi} 2\langle\overline{\mathbf{D}}\rangle^v - \frac{2}{3}\phi\rho\langle k\rangle^i\mathbf{I} \tag{26}$$

where

$$\langle\overline{\mathbf{D}}\rangle^v = \frac{1}{2}\{\nabla(\phi\langle\overline{\mathbf{u}}\rangle^i) + [\nabla(\phi\langle\overline{\mathbf{u}}\rangle^i)]^T\} \tag{27}$$

is the macroscopic deformation tensor, $\langle k\rangle^i$ is the intrinsic average for k and μ_{t_ϕ} is the macroscopic turbulent viscosity assumed to be in [15] as,

$$\mu_{t_\phi} = \rho c_\mu \frac{\langle k\rangle^{i\,2}}{\langle\varepsilon\rangle^i} \tag{28}$$

Rocamora and de Lemos [18], following a similar path, applied (11) to the time-averaged microscopic energy equations for both the fluid and the solid matrix. With this procedure, two new equations arose, one for the time-mean intrinsic fluid temperature, $\langle\overline{T_f}\rangle^i$, and other for the solid temperature $\langle\overline{T_s}\rangle^i$. Further, assuming the Local Thermal Equilibrium Hypothesis ($\langle\overline{T_f}\rangle^i = \langle\overline{T_s}\rangle^i = \langle\overline{T}\rangle^i$) and adding up these two equations, a unique macroscopic energy equation was obtained. A proposed form for this equation is given in Rocamora and de Lemos [19] as,

$$(\rho c_p)_f\,\nabla\cdot(\mathbf{u}_D\langle\overline{T}\rangle^i) = \nabla\cdot\{\mathbf{K}_{eff}\cdot\nabla\langle\overline{T}\rangle^i\} \tag{29}$$

where $\mathbf{K}_{eff}$ is the effective conductivity tensor given by,

$$\mathbf{K}_{eff} = [\phi k_f + (1-\phi)k_s]\mathbf{I} + \mathbf{K}_{tor} + \mathbf{K}_t + \mathbf{K}_{disp} + \mathbf{K}_{disp,t} \tag{30}$$

In (30), k_f and k_s are the fluid and solid thermal conductivities, respectively, and the **K**'s are components of $\mathbf{K}_{eff}$ due to the tortuosity ($\mathbf{K}_{tor}$), thermal dispersion ($\mathbf{K}_{disp}$), turbulent heat flux ($\mathbf{K}_t$) and turbulent thermal dispersion ($\mathbf{K}_{disp,t}$).

In order to apply (29), it is necessary to determine the conductivity tensor components in (30), *i.e.*, $\mathbf{K}_{tor}$, $\mathbf{K}_t$, $\mathbf{K}_{disp}$ and $\mathbf{K}_{disp,t}$. Following the work in [8], this can be accomplished for the tortuosity and thermal dispersion conductivity tensors, $\mathbf{K}_{tor}$ and $\mathbf{K}_{disp}$, by making use of a unit cell subjected to periodic boundary conditions for the flow and by imposing a linear temperature gradient over the domain. The conductivity tensors are then obtained directly from the microscopic results for this unit cell (see [8] for details).

On the other hand, the turbulent heat flux and turbulent thermal dispersion terms, $\mathbf{K}_t$ and $\mathbf{K}_{disp,t}$, cannot be determined from such a microscopic calculation and were modeled in reference [19] through the Eddy Diffusivity concept, similarly to the work in reference [9]. It should be noticed that these terms arise only if the flow is turbulent, whereas the *tortuosity* and the *thermal dispersion* terms exist for both laminar and turbulent flow regimes. Thus, the model proposed in [19] for the macroscopic turbulent heat flux follows the concept embodied in (21) and reads,

$$-(\rho c_p)_f \langle \overline{\mathbf{u}' T_f'} \rangle^i = c_{p_f} \frac{\mu_{t_\phi}}{\sigma_{T_\phi}} \nabla \langle \overline{T}_f \rangle^i \tag{31}$$

where μ_{t_ϕ} is given by (28), σ_{T_ϕ} is a constant and the subscript *f*, as before, identifies fluid phase properties. According to equation (31), the macroscopic turbulent heat flux is taken as the sum of the turbulent heat flux and the turbulent thermal dispersion, as proposed in [18]. These two terms were related there to the components of the conductivity tensor, $\mathbf{K}_t$ and $\mathbf{K}_{disp,t}$, respectively, by the expression,

$$\mathbf{K}_t + \mathbf{K}_{disp,t} = \phi\, c_{p_f} \frac{\mu_{t_\phi}}{\sigma_{T_\phi}} \mathbf{I} \tag{32}$$

Macroscopic Buoyancy Effects

Mean Flow

Focusing now attention to buoyancy effects only, application of the volume average procedure to the last term of (18) leads to,

$$\langle \rho \mathbf{g} \beta (\overline{T} - T_{ref}) \rangle^v = \frac{\Delta V_f}{\Delta V} \frac{1}{\Delta V_f} \int_{\Delta V_f} \rho \mathbf{g} \beta (\overline{T} - T_{ref})\, dV \tag{33}$$

Expanding the left hand side of (33) in light of (10), the buoyancy term becomes,

$$\langle \rho \mathbf{g} \beta (\overline{T} - T_{ref}) \rangle^v = \rho \beta_\phi \mathbf{g} \phi (\langle \overline{T} \rangle^i - T_{ref}) + \underbrace{\rho \mathbf{g} \beta \phi \overline{\langle {}^i T \rangle^i}}_{=0} \tag{34}$$

where the second term on the r.h.s. is null since $\langle {}^i \varphi \rangle^i = 0$. Here, the coefficient β_ϕ is the macroscopic thermal expansion coefficient. Assuming that gravity is constant over the REV, an expression for it based on (34) is given as,

$$\beta_\phi = \frac{\langle \rho \beta (\overline{T} - T_{ref}) \rangle^v}{\rho \phi (\langle \overline{T} \rangle^i - T_{ref})} \tag{35}$$

Including (34) into (25), the macroscopic time-mean Navier-Stokes (NS) equation for an incompressible fluid with constant properties is given as,

$$\rho \nabla \cdot \left(\frac{\overline{\mathbf{u}}_D \overline{\mathbf{u}}_D}{\phi} \right) = -\nabla (\phi \langle \overline{p} \rangle^i) + \mu \nabla^2 \overline{\mathbf{u}}_D + \nabla \cdot (-\rho \phi \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i) + \rho \beta_\phi \mathbf{g} \phi (\langle \overline{T} \rangle^i - T_{ref}) - \left[\frac{\mu \phi}{K} \overline{\mathbf{u}}_D + \frac{c_F \phi \rho |\overline{\mathbf{u}}_D| \, \overline{\mathbf{u}}_D}{\sqrt{K}} \right] \tag{36}$$

Turbulent Field

As mentioned, this work extends the development in Pedras and de Lemos [15] in order to include the buoyancy production rate term in the turbulence model equations. For clear flows, the buoyancy contribution to the k equation is given by equation (23). Applying the volume average operator to that term, one has,

$$\langle G_k \rangle^v = G^i_\beta = \langle -\rho \beta \mathbf{g} \cdot \overline{\mathbf{u}'T'} \rangle^v = -\rho \beta^k_\phi \phi \mathbf{g} \cdot \langle \overline{\mathbf{u}'T'_f} \rangle^i \tag{37}$$

where the coefficient β^k_ϕ, for a constant value of $\mathbf{g}$ within the REV, is given by $\beta^k_\phi = \dfrac{\langle \beta \overline{\mathbf{u}'T'} \rangle^v}{\phi \langle \overline{\mathbf{u}'T'_f} \rangle^i}$, which, in turn, is not necessarily equal to β_ϕ given by (35). However, for the sake of simplicity and in the absence of better information, one can make use of the assumption $\beta^k_\phi = \beta_\phi = \beta$. Further, expanding the r.h.s. of (37) in light of (10) and (14), one has

$$\begin{aligned} -\rho \beta^k_\phi \phi \mathbf{g} \cdot \langle \overline{\mathbf{u}'T'_f} \rangle^i &= -\rho \beta^k_\phi \phi \mathbf{g} \cdot \langle \overline{(\langle \mathbf{u}' \rangle^i + {}^i\mathbf{u}')(\langle T'_f \rangle^i + {}^iT'_f)} \rangle^i \\ &= -\rho \beta^k_\phi \phi \mathbf{g} \cdot \left(\langle \overline{\langle \mathbf{u}' \rangle^i \langle T'_f \rangle^i} \rangle^i + \langle \overline{{}^i\mathbf{u}' \, {}^iT'_f} \rangle^i + \langle \overline{\langle \mathbf{u}' \rangle^i \, {}^iT'_f} \rangle^i + \langle \overline{{}^i\mathbf{u}' \langle T'_f \rangle^i} \rangle^i \right) \\ &= -\rho \beta^k_\phi \phi \mathbf{g} \cdot \left(\underbrace{\overline{\langle \mathbf{u} \rangle^{i\,\prime} \langle T_f \rangle^{i\,\prime}}}_{\text{I}} + \underbrace{\langle \overline{{}^i\mathbf{u}' \, {}^iT'_f} \rangle^i}_{\text{II}} + \underbrace{\overline{\langle \mathbf{u}' \rangle^i \langle {}^iT'_f \rangle^i}}_{=0} + \underbrace{\overline{\langle {}^i\mathbf{u}' \rangle^i \langle T'_f \rangle^i}}_{=0} \right) \end{aligned} \tag{38}$$

The last two terms on the right of (38) are null since $\langle {}^iT'_f \rangle^i = 0$ and $\langle {}^i\mathbf{u}' \rangle^i = 0$. In addition, the following physical significance can be inferred to the two remaining:

I. **Generation/destruction rate due to macroscopic time fluctuations**: Buoyancy generation/destructions rate of $\langle k \rangle^i$ due to time fluctuations of macroscopic velocity and temperature. This term is also present in turbulent flow in clear (non-obstructed) domains and represents an exchange between the energy associated with the macroscopic turbulent motion and potential energy. In stable stratification, this term damps turbulence by being of negative value whereas the potential energy of the system is increased. On the other hand, in unstable stratification, it enhances $\langle k\rangle^i$ at the expense of potential energy.

II. **Generation/destruction rate due to turbulent buoyant dispersion**: Buoyancy generation/destruction rate of $\langle k\rangle^i$ in a porous medium due to time fluctuations and spatial deviations of both microscopic velocity and temperature. This term might be interpreted as an additional source/sink of turbulence kinetic energy due the fact that time fluctuations of local velocities and temperatures present a spatial deviation in relation to their macroscopic value. Then, additional exchange between turbulent kinetic energy and potential energy in systems may occur due to the presence of a porous matrix.

A model for (38) is still needed in order to solve an equation for $\langle k\rangle^i$, which is a necessary information when computing μ_{t_ϕ} using (28). Consequently, terms I and II above have to be modeled as a function of average temperature, $\langle \overline{T}\rangle^i$. To accomplish this, a gradient type diffusion model is used, in the form,

- Buoyancy generation of $\langle k\rangle^i$ due to **turbulent fluctuations**:

$$-\rho\beta_\phi^k\phi\mathbf{g}\cdot\overline{\langle\mathbf{u}\rangle^{i'}\langle T_f\rangle^{i'}}=\rho\mathbf{B}_t\cdot\nabla\langle\overline{T}\rangle^i \tag{39}$$

- Buoyancy generation of $\langle k\rangle^i$ due to **turbulent buoyant dispersion**:

$$-\rho\beta_\phi^k\phi\mathbf{g}\cdot\langle\overline{{}^i\mathbf{u}'\,{}^iT_f'}\rangle^i=\rho\mathbf{B}_{disp,t}\cdot\nabla\langle\overline{T}\rangle^i \tag{40}$$

The buoyancy coefficients seem above, namely $\mathbf{B}_t$ and $\mathbf{B}_{disp,t}$, are modeled here through the Eddy Diffusivity concept, similarly to the work in [9]. It should be noticed that these terms arise only if the flow is turbulent and if buoyancy is of importance.

Using then an expression similar to (31), the macroscopic buoyancy generation of k can be modeled as,

$$G_\beta^i=-\rho\beta_\phi^k\,\phi\,\mathbf{g}\cdot\langle\overline{\mathbf{u}'T_f'}\rangle^i=\beta_\phi^k\phi\frac{\mu_{t_\phi}}{\sigma_{T_\phi}}\mathbf{g}\cdot\nabla\langle\overline{T}\rangle^i=\mathbf{B}_{eff}\cdot\nabla\langle\overline{T}\rangle^i \tag{41}$$

where μ_{t_ϕ} and σ_{T_ϕ} have been defined before and the two coefficients $\mathbf{B}_t$ and $\mathbf{B}_{disp,t}$ are expressed as,

$$\mathbf{B}_t+\mathbf{B}_{disp,t}=\mathbf{B}_{eff}=\beta_\phi^k\phi\frac{\mu_{t_\phi}}{\sigma_{T_\phi}}\mathbf{g} \tag{42}$$

Final transport equations for $\langle k\rangle^i = \langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2$ and $\langle\varepsilon\rangle^i = \mu\langle\overline{\nabla\mathbf{u}':(\nabla\mathbf{u}')^T}\rangle^i/\rho$, in their so-called High Reynolds number form, as proposed in Pedras and de Lemos [15], can now include the buoyancy generation terms seen above as,

$$\rho\nabla\cdot\left(\overline{\mathbf{u}}_D\langle k\rangle^i\right)=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_k}\right)\nabla(\phi\langle k\rangle^i)\right]+P^i+G^i+G^i_\beta-\rho\phi\langle\varepsilon\rangle^i \tag{43}$$

$$\rho\nabla\cdot\left(\overline{\mathbf{u}}_D\langle\varepsilon\rangle^i\right)=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_\varepsilon}\right)\nabla\left(\phi\langle\varepsilon\rangle^i\right)\right]+\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i}\left[c_1P^i+c_2G^i+c_1c_3G^i_\beta-c_2\rho\phi\langle\varepsilon\rangle^i\right] \tag{44}$$

where c_1, c_2, c_3 and c_k are constants, $P^i=-\rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i:\nabla\overline{\mathbf{u}}_D$ is the production rate of $\langle k\rangle^i$ due to gradients of $\overline{\mathbf{u}}_D$, $G^i=c_k\rho\frac{\phi\langle k\rangle^i|\overline{\mathbf{u}}_D|}{\sqrt{K}}$ is the generation rate of the intrinsic average of $\langle k\rangle^i$ due to the action of the porous matrix and $G^i_\beta=\mathbf{B}_{eff}\cdot\nabla\langle\overline{T}\rangle^i$ is the generation of $\langle k\rangle^i$ due to buoyancy.

Conclusions

This work presented, under the light of the double decomposition concept [14,15], the derivation of the buoyancy terms in the equations for turbulent natural convection in saturated rigid porous media. The turbulent kinetic energy generation/destruction rate, own to buoyancy effects, was split into two terms, one related to macroscopic **turbulent fluctuations**, and other due to a new mechanism here named **turbulent buoyant dispersion**. A possible interpretation for this second term was an additional exchange mechanism between turbulent kinetic energy and potential energy in thermal systems, which may occur due to the presence of a porous matrix. Complete macroscopic equations for k and we r e then presented. Analyses of important environmental and engineering flows can benefit from the derivations herein and, ultimately, it is expected that additional research on this new subject be stimulated by the work here presented.

Acknowledgments

The authors would like to thank CNPq, Brazil, for their financial support during the preparation of this work.

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Received November 26, 2002