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ANALYSIS OF CONVECTIVE HEAT TRANSFER FOR TURBULENT FLOW IN SATURATED POROUS MEDIA

Francisco D. Rocamora Jr.
Instituto de Estudos Avançados – IEAv/CTA
12228-900 - São José dos Campos, SP - Brazil

Marcelo J.S. de Lemos*
Departamento de Energia, IEME
Instituto Tecnológico de Aeronáutica – ITA/CTA
12228-900 - São José dos Campos, SP – Brazil
*Corresponding Author. E-mail: delemos@mec.ita.br

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ABSTRACT

The literature documents two procedures for modeling turbulent heat transport in incompressible flows through homogeneous rigid porous media. The first method considers time averaging of the energy equation before the volume average operator is applied. The second methodology also employs both averaging operators, but in the reverse order. Resulting equations in both cases are different, leading to controversies and interesting discussions in the literature. This work is intended to demonstrate that both approaches lead to equivalent equations when one takes into account both *time fluctuations* and *spatial deviations* of velocity and temperature. © 2000 Elsevier Science Ltd

Introduction

Modeling of turbulent heat transport for incompressible flows in rigid porous media is based in one of the two following methodologies, both applied to the instantaneous local energy equation: *a*) application of time-average operator followed by volume-averaging [1-4], and *b*) use of volume-averaging before time-averaging is applied [5-7]. These two different methodologies give rise to different sets of transport equations and, consequently, to interesting discussions in the literature. This work is intended to show that both sets of macroscopic energy equations are equivalent when examined under the recently established *double decomposition*

concept [8-9]. In this methodology, both time fluctuations and spatial deviation are considered for velocity and temperature.

Local Instantaneous Energy Equation

The energy equation for steady-state incompressible flow can be stated as:

$$(\rho c_p)\nabla\cdot(\mathbf{u}T)=\nabla\cdot(k\nabla T)+S_T \tag{1}$$

where ρ is the density, c_p is the specific heat, $\mathbf{u}$ is the velocity, T is the temperature, k is the thermal conductivity and S_T is the source term (heat generation rate per unit volume).

For saturated rigid porous media, the energy equations for the fluid and solid phases can then be written as,

- Fluid -

$$(\rho c_p)_f\nabla\cdot(\mathbf{u}T_f)=\nabla\cdot(k_f\nabla T_f)+S_f \tag{2}$$

- Solid - (Porous Matrix)

$$0=\nabla\cdot(k_s\nabla T_s)+S_s \tag{3}$$

where the subscripts f and s refer to fluid and solid phases, respectively. If there is no heat generation either in the solid or in the fluid one has further,

$$S_f=S_s=0 \tag{4}$$

As mentioned, there are, in principle, two ways that one can follow to treat turbulent flow in porous media. The first method applies a time average operator to the governing equation (1) before the volume average procedure is conducted. In the second approach, the order of application of the two average operators is reversed. Both techniques aim at derivation of a suitable macroscopic energy equation.

Volume averaging in a porous medium, described in references [10,11,12], makes use of the concept of a Representative Elementary Volume (REV), over which local equations are integrated. After integration, detailed information within the volume is lost and, instead, overall properties referring to a REV are considered. In a similar manner, statistical analysis of turbulent flow leads to time mean properties. Transport equations for statistical values are considered in lieu of instantaneous information on the flow.

Before undertaking the task of developing macroscopic equations, it is convenient to recall the definition of time average and volume average.

Time Average and Volume Average

The time average of a general quantity φ is defined as:

$$\overline{\varphi} = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} \varphi \, dt \tag{5}$$

where the time interval Δt is small compared to the fluctuations of the average value, $\overline{\varphi}$, but large enough to capture turbulent fluctuations of φ. Time decomposition can then be written as,

$$\varphi = \overline{\varphi} + \varphi' \tag{6}$$

with $\overline{\varphi'} = 0$. Here, φ' is the *time fluctuation* of φ around its average $\overline{\varphi}$.

The volume average of φ taken over a Representative Elementary Volume in a porous medium can be written as:

$$\langle \varphi \rangle^{v} = \frac{1}{\Delta V}\int_{\Delta V} \varphi \, dV \tag{7}$$

The value $\langle \varphi \rangle^{v}$ is defined for any point **x** surrounded by a Representative Elementary Volume, of size ΔV. This average is related to the *intrinsic* average for the fluid phase as:

$$\langle \varphi_f \rangle^{v} = \phi \langle \varphi_f \rangle^{i} \tag{8}$$

where $\phi = \Delta V_f / \Delta V$ is the medium porosity and ΔV_f is the volume occupied by the fluid in a REV. Furthermore, one can write:

$$\varphi = \langle \varphi \rangle^{i} + {}^{i}\varphi \tag{9}$$

with $\langle {}^{i}\varphi \rangle^{i} = 0$. In equation (9), ${}^{i}\varphi$ is the *spatial deviation* of φ with respect to the intrinsic average $\langle \varphi \rangle^{i}$.

Further, the local volume average theorem can be expressed as [10,11,12]:

$$\langle \nabla \cdot \varphi \rangle^{v} = \nabla \cdot \langle \varphi \rangle^{v} + \frac{1}{\Delta V}\int_{A_i} \mathbf{n} \cdot \varphi \, dS \tag{10}$$

$$\langle \nabla \varphi \rangle^{v} = \nabla \langle \varphi \rangle^{v} + \frac{1}{\Delta V}\int_{A_i} \mathbf{n} \varphi \, dS \tag{11}$$

where **n** is the unit vector normal to the fluid-solid interface, and A_i is the fluid-solid interface area within the REV. It is important to emphasize that A_i should not be confused with the surface area surrounding volume ΔV.

Time Average followed by Volume Average

In order to apply the time average operator to equations (2) and (3), one considers,

$$T = \overline{T} + T' \tag{12}$$

$$\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}' \tag{13}$$

Substituting (12) and (13) into (2) and (3), respectively, one has:

$$(\rho c_p)_f \nabla \cdot (\overline{\mathbf{u}}\,\overline{T_f} + \overline{\mathbf{u}} T_f' + \mathbf{u}'\overline{T_f} + \mathbf{u}' T_f') = \nabla \cdot (k_f \nabla(\overline{T_f} + T_f')) \tag{14}$$

$$0 = \nabla \cdot (k_s \nabla(\overline{T_s} + T_s')) \tag{15}$$

Applying time average to (14) and (15), one obtains:

$$(\rho c_p)_f \nabla \cdot (\overline{\mathbf{u}}\,\overline{T_f} + \overline{\mathbf{u}' T_f'}) = \nabla \cdot (k_f \nabla \overline{T_f}) \tag{16}$$

$$0 = \nabla \cdot (k_s \nabla \overline{T_s}) \tag{17}$$

The second term on the left of (16) is known as **turbulent heat flux**. It requires a model for closure of the mathematical problem. Also, in order to apply the volume average to (16) and (17), one must first define the spatial deviations with respect to the time averages, given by:

$$\overline{T} = \langle \overline{T} \rangle^{\iota} + {}^{\iota}\overline{T} \tag{18}$$

$$\overline{\mathbf{u}} = \langle \overline{\mathbf{u}} \rangle^{\iota} + {}^{\iota}\overline{\mathbf{u}} \tag{19}$$

Substituting now (18) and (19) into (14) and (15), respectively, and performing the volume average operation, one has:

$$\begin{aligned}(\rho c_p)_f \nabla \cdot \{\phi(\langle \overline{\mathbf{u}} \rangle^{\iota} \langle \overline{T_f} \rangle^{\iota} + \langle {}^{\iota}\overline{\mathbf{u}}\, {}^{\iota}\overline{T_f} \rangle^{\iota} + \langle \overline{\mathbf{u}' T_f'} \rangle^{\iota})\} &= \nabla \cdot [k_f \nabla(\phi \langle \overline{T_f} \rangle^{\iota})] \\ &+ \nabla \cdot \left[\frac{1}{\Delta V}\int_{A_i} \mathbf{n} k_f \overline{T_f}\, dS\right] + \frac{1}{\Delta V}\int_{A_i} \mathbf{n} \cdot k_f \nabla \overline{T_f}\, dS\end{aligned} \tag{20}$$

$$\nabla \cdot \{k_s \nabla[(1-\phi)\langle \overline{T_s} \rangle^{\iota}]\} - \nabla \cdot \left[\frac{1}{\Delta V}\int_{A_i} \mathbf{n} k_s \overline{T_s}\, dS\right] - \frac{1}{\Delta V}\int_{A_i} \mathbf{n} \cdot k_s \nabla \overline{T_s}\, dS = 0 \tag{21}$$

Equations (20) and (21) are the macroscopic energy equations for the fluid and the porous matrix (solid) taking first the *time average* followed by the *volume average* operator.

Volume Average followed by Time Average

To apply the volume average to (2) and (3) one has:

$$T = \langle T \rangle^{\iota} + {}^{\iota}T \tag{22}$$

$$\mathbf{u} = \langle \mathbf{u} \rangle^{\iota} + {}^{\iota}\mathbf{u} \tag{23}$$

in addition,

$$\left.\begin{aligned}\langle T\rangle^{v} &= \gamma\langle T\rangle^{i}\\ \langle \mathbf{u}\rangle^{v} &= \gamma\langle \mathbf{u}\rangle^{i}\end{aligned}\right\} \quad \text{where} \quad \gamma = \begin{cases}\phi & \textit{for the fluid}\\ (1-\phi) & \textit{for the solid}\end{cases} \tag{24}$$

Substituting (22) and (23) into (2) and (3), one obtains:

$$(\rho c_p)_f \nabla\cdot(\langle\mathbf{u}\rangle^{i}\langle T_f\rangle^{i} + \langle\mathbf{u}\rangle^{i\,\prime}T_f + {}^{\prime}\mathbf{u}\langle T_f\rangle^{i} + {}^{\prime}\mathbf{u}\,{}^{\prime}T_f) = \nabla\cdot[k_f\nabla(\langle T_f\rangle^{i} + {}^{\prime}T_f)] \tag{25}$$

$$0 = \nabla\cdot[k_s\nabla(\langle T_s\rangle^{i} + {}^{\prime}T_s)] \tag{26}$$

Taking the volume average of (25) and (26), one has:

$$\begin{aligned}(\rho c_p)_f \nabla\cdot[\phi(\langle\mathbf{u}\rangle^{i}\langle T_f\rangle^{i} + \langle{}^{\prime}\mathbf{u}\,{}^{\prime}T_f\rangle^{i})] &= \nabla\cdot[k_f\nabla(\phi\langle T_f\rangle^{i})]\\ &+\nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}k_f T_f\,dS\right] + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot k_f\nabla T_f\,dS\end{aligned} \tag{27}$$

$$\nabla\cdot\{k_s\nabla[(1-\phi)\langle T_s\rangle^{i}]\} - \nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}k_s T_s\,dS\right] - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot k_s\nabla T_s\,dS = 0 \tag{28}$$

The second term on the left of (27) appears in classical analysis of convection in porous media (*e.g.* [13]) and is known as **thermal dispersion**. In order to apply the time average to (27) and (28), one defines the intrinsic volume average as:

$$\langle T\rangle^{i} = \overline{\langle T\rangle^{i}} + \langle T\rangle^{i\,\prime} \tag{29}$$

$$\langle\mathbf{u}\rangle^{i} = \overline{\langle\mathbf{u}\rangle^{i}} + \langle\mathbf{u}\rangle^{i\,\prime} \tag{30}$$

Substituting (29) and (30) in (27) and (28) and taking the time average, we obtain:

$$\begin{aligned}(\rho c_p)_f \nabla\cdot\left[\phi\left(\overline{\langle\mathbf{u}\rangle^{i}\langle T_f\rangle^{i}} + \overline{\langle\mathbf{u}\rangle^{i\,\prime}\langle T_f\rangle^{i\,\prime}} + \overline{\langle{}^{\prime}\mathbf{u}\,{}^{\prime}T_f\rangle^{i}}\right)\right] &= \nabla\cdot[k_f\nabla(\phi\overline{\langle T_f\rangle^{i}})]\\ &+\nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}k_f\overline{T_f}\,dS\right] + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot k_f\nabla\overline{T_f}\,dS\end{aligned} \tag{31}$$

$$\nabla\cdot\{k_s\nabla[(1-\phi)\overline{\langle T_s\rangle^{i}}]\} - \nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}k_s\overline{T_s}\,dS\right] - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot k_s\nabla\overline{T_s}\,dS = 0 \tag{32}$$

Equations (31) and (32) are the macroscopic energy equations for the fluid and the porous matrix (solid) taking first the *volume average* followed by the *time average*.

It is interesting to observe that (20) and (21), obtained through the first procedure (*time-volume* average) are similar to (31) and (32), respectively, obtained through the second method (*volume-time* average). To show their equivalence is the purpose of next section.

The Double Decomposition Concept

Pedras and de Lemos [8,9] showed that for a rigid, homogeneous porous medium saturated with an incompressible fluid, the following relationships apply:

$$\overline{\langle\varphi\rangle^{i}} = \langle\overline{\varphi}\rangle^{i} \tag{33}$$

i.e., the time and volume averages commute. Also,

$$\left.\begin{aligned} {}^{i}\overline{\varphi} &= \overline{{}^{i}\varphi} \\ \langle\varphi'\rangle^{i} &= \langle\varphi\rangle^{i\,\prime} \end{aligned}\right. \tag{34}$$

and

$$\left.\begin{aligned} \varphi' &= \langle\varphi'\rangle^{i} + {}^{i}\varphi' \\ {}^{i}\varphi &= \overline{{}^{i}\varphi} + {}^{i}\varphi' \end{aligned}\right\} \quad \text{where} \quad {}^{i}\varphi' = \varphi' - \langle\varphi'\rangle^{i} = {}^{i}\varphi - \overline{{}^{i}\varphi} \tag{35}$$

Therefore, the quantity φ can be expressed by either,

$$\varphi = \overline{\langle\varphi\rangle^{i}} + \langle\varphi\rangle^{i\,\prime} + \overline{{}^{i}\varphi} + {}^{i}\varphi' \tag{36}$$

or

$$\varphi = \langle\overline{\varphi}\rangle^{i} + {}^{i}\overline{\varphi} + \langle\varphi'\rangle^{i} + {}^{i}\varphi' \tag{37}$$

Expressions (36) and (37) comprise the double decomposition concept where ${}^{i}\varphi'$ can be understood as either the *time fluctuation of the spatial deviation* or the *spatial deviation of the time fluctuation*. Also, $\langle{}^{i}\varphi'\rangle^{i} = \overline{{}^{i}\varphi'} = 0$.

With the help of Figure 1, the concept of *double decomposition* proposed in [8,9] can be visualized. The Figure shows a three-dimensional diagram for a general vector variable φ. For a scalar, all the quantities shown would be drawn on a single line. Also, notice that points ***B***, ***C***, ***D*** and ***E*** fall in the same plane, with segments ***BC*** and ***BE*** parallel to ***ED*** and ***CD***, respectively. Line ***AEF*** represents standard time decomposition given by (6) whereas equation (9) is pictured by line ***ACF***. Further, (19) is given by line ***ABE*** and (30) by segment ***ABC***. Equality (33) is represented by ***AB*** and the two equations in (34) by the equivalence between the parallel segments ***BE*** and ***CD*** and between ***BC*** and ***ED***. Triangles ***EDF*** and ***CDF*** are associated with expression (35). Finally, equation (36) follow the sequence ***ABCDE*** and relationship (37) is represented by the path ***ABEDF***, both of them decomposing the same general variable φ.

Using now (34) and (35), the third terms on the left hand side of (20) and (31) can be expanded as:

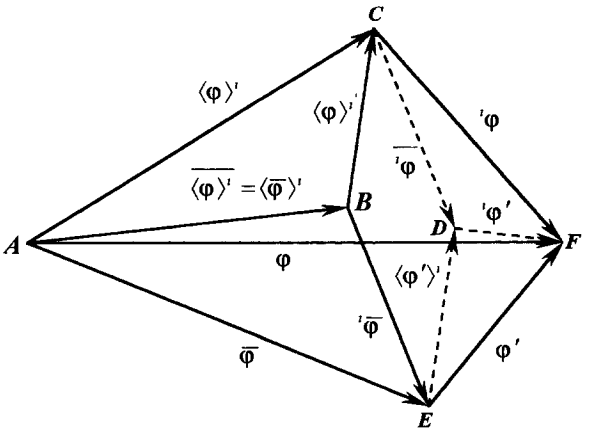

FIG. 1
General three-dimensional vector diagram for a quantity φ.

$$\langle \overline{\mathbf{u}' T_f'} \rangle' = \langle \overline{(\langle \mathbf{u}' \rangle' + {}' \mathbf{u}')(\langle T_f' \rangle' + {}' T')} \rangle' = \overline{\langle \mathbf{u}' \rangle' \langle T_f' \rangle'} + \langle \overline{{}' \mathbf{u}' \, {}' T_f'} \rangle' \tag{38}$$

$$\overline{\langle {}' \mathbf{u} \, {}' T_f \rangle'} = \overline{\langle ({}' \overline{\mathbf{u}} + {}' \mathbf{u}')({}' \overline{T_f} + {}' T') \rangle'} = \langle {}' \overline{\mathbf{u}} \, {}' \overline{T_f} \rangle' + \overline{\langle {}' \mathbf{u}' \, {}' T_f' \rangle'} \tag{39}$$

Substituting (38) into (20), the convection term will read,

$$(\rho c_p)_f \nabla \cdot (\overline{\phi \langle \mathbf{u} \, T \rangle'}) = (\rho c_p)_f \nabla \cdot \{\phi (\langle \overline{\mathbf{u}} \rangle' \langle \overline{T_f} \rangle' + \langle {}' \overline{\mathbf{u}} \, {}' \overline{T_f} \rangle' + \overline{\langle \mathbf{u}' \rangle' \langle T_f' \rangle'} + \langle \overline{{}' \mathbf{u}' \, {}' T_f'} \rangle')\} \tag{40}$$

Also, plugging (39) into (31) will give for the same convection term,

$$(\rho c_p)_f \nabla \cdot (\overline{\phi \langle \mathbf{u} \, T \rangle'}) = (\rho c_p)_f \nabla \cdot \{\phi (\overline{\langle \mathbf{u} \rangle' \langle T_f \rangle'} + \overline{\langle \mathbf{u} \rangle' \langle T_f \rangle'}' + \langle {}' \overline{\mathbf{u}} \, {}' \overline{T_f} \rangle' + \langle \overline{{}' \mathbf{u}' \, {}' T_f'} \rangle')\} \tag{41}$$

$$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$$
$$\mathbf{I} \quad \mathbf{II} \quad \mathbf{III} \quad \mathbf{IV}$$

Comparing (40) to (41), in light of (33) and (34), one can conclude that (20) and (21) are, in fact, equal to (31) and (32), respectively. This demonstrates that the final expanded form of the macroscopic energy equation for a rigid, homogeneous porous medium saturated with an incompressible fluid, does not depend on the averaging order, *i.e.*, both procedures lead to the same results.

Further, the four terms on the right of (41) could be given the following physical significance:

I. **Convective heat flux** based on macroscopic time mean velocity and temperature.
II. **Turbulent heat flux** due to the fluctuating components of macroscopic velocity and temperature.

III. **Thermal dispersion** associated with deviations of microscopic time mean velocity and temperature. Note that this term is also present when analyzing laminar convective heat transfer in porous media.

IV. **Turbulent thermal dispersion** in a porous medium due to both time fluctuations and spatial deviations of both microscopic velocity and temperature.

Thus, the macroscopic energy equations for an incompressible flow in a rigid, homogeneous and saturated porous medium can be written as:

-Fluid-

$$(\rho c_p)_f \nabla\cdot[\phi(\langle\overline{\mathbf{u}}\rangle^{'}\langle\overline{T_f}\rangle^{'}+\langle^{'}\overline{\mathbf{u}}^{'}\overline{T_f}\rangle^{'}+\overline{\langle\mathbf{u}'\rangle^{'}\langle T_f'\rangle^{'}}+\langle\overline{^{'}\mathbf{u}'\,^{'}T_f'}\rangle^{'})]=$$
$$\nabla\cdot[k_f\nabla(\phi\langle\overline{T_f}\rangle^{'})]+\nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}k_f\overline{T_f}\,dS\right]+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot k_f\nabla\overline{T_f}\,dS \tag{42}$$

-Solid (Porous Matrix)-

$$\nabla\cdot\{k_s\nabla[(1-\phi)\langle\overline{T_s}\rangle^{'}]\}-\nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}k_s\overline{T_s}\,dS\right]-\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot k_s\nabla\overline{T_s}\,dS=0 \tag{43}$$

Further, adding (42) to (43), a global macroscopic energy equation can be obtained as:

$$(\rho c_p)_f \nabla\cdot[\phi(\langle\overline{\mathbf{u}}\rangle^{'}\langle\overline{T_f}\rangle^{'}+\langle^{'}\overline{\mathbf{u}}^{'}\overline{T_f}\rangle^{'}+\overline{\langle\mathbf{u}'\rangle^{'}\langle T_f'\rangle^{'}}+\langle\overline{^{'}\mathbf{u}'\,^{'}T_f'}\rangle^{'})]=$$
$$\nabla\cdot\{k_f\nabla(\phi\langle\overline{T_f}\rangle^{'})+k_s\nabla[(1-\phi)\langle\overline{T_s}\rangle^{'}]\}+$$
$$\nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}(k_f\overline{T_f}-k_s\overline{T_s})dS\right]+ \tag{44}$$
$$\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot(k_f\nabla\overline{T_f}-k_s\nabla\overline{T_s})dS$$

where the applicable boundary conditions on the surface A_i are given by:

$$\left.\begin{aligned}T_f&=T_s\\ \mathbf{n}\cdot(k_f\nabla T_f)&=\mathbf{n}\cdot(k_s\nabla T_s)\end{aligned}\right\}\ in\ A_i \tag{45}$$

In view of the boundary conditions expressed by (45), one verifies that the last term on the right hand side of (44) vanishes (due to the heat flux continuity at the fluid-solid interface). Thus, one can write:

$$(\rho c_p)_f \nabla\cdot[\phi(\langle\overline{\mathbf{u}}\rangle^{'}\langle\overline{T_f}\rangle^{'}+\langle^{'}\overline{\mathbf{u}}^{'}\overline{T_f}\rangle^{'}+\overline{\langle\mathbf{u}'\rangle^{'}\langle T_f'\rangle^{'}}+\langle\overline{^{'}\mathbf{u}'\,^{'}T_f'}\rangle^{'})]=$$
$$\nabla\cdot\{k_f\nabla[\phi\langle\overline{T_f}\rangle^{'}]+k_s\nabla[(1-\phi)\langle\overline{T_s}\rangle^{'}]\}+\nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}(k_f\overline{T_f}-k_s\overline{T_s})dS\right] \tag{46}$$

Local Thermal Equilibrium Hypothesis

The local thermal equilibrium hypothesis assumes that the intrinsic average of the time mean temperature for fluid and solid phases are equal, or say,

$$\langle \overline{T_f} \rangle^i = \langle \overline{T_s} \rangle^i = \langle \overline{T} \rangle^i \tag{47}$$

Thus, applying (47) in (46) one gets,

$$\begin{aligned}(\rho c_p)_f \nabla \cdot (\phi \langle \overline{\mathbf{u}} \rangle^i \langle \overline{T} \rangle^i) = \nabla \cdot \{[k_f \phi + k_s (1-\phi)] \nabla \langle \overline{T} \rangle^i\} + \\ \nabla \cdot \left[\frac{1}{\Delta V} \int_{A_i} \mathbf{n} (k_f \overline{T_f} - k_s \overline{T_s}) dS \right] - \\ (\rho c_p)_f \nabla \cdot \{\phi (\langle {}^i \overline{\mathbf{u}}^i \overline{T_f} \rangle^i + \overline{\langle \mathbf{u}' \rangle^i \langle T_f' \rangle^i} + \langle \overline{{}^i \mathbf{u}'^i T_f'} \rangle^i)\}\end{aligned} \tag{48}$$

Using further the Dupuit-Forchheimer relationship $\mathbf{u}_D = \langle \overline{\mathbf{u}} \rangle^v = \phi \langle \overline{\mathbf{u}} \rangle^i$, one can rewrite (48) as:

$$\begin{aligned}(\rho c_p)_f \nabla \cdot (\mathbf{u}_D \langle \overline{T} \rangle^i) = \nabla \cdot \{[k_f \phi + k_s (1-\phi)] \nabla \langle \overline{T} \rangle^i\} + \\ \nabla \cdot \left[\frac{1}{\Delta V} \int_{A_i} \mathbf{n} (k_f \overline{T_f} - k_s \overline{T_s}) dS \right] - \\ (\rho c_p)_f \nabla \cdot [\phi (\langle {}^i \overline{\mathbf{u}}^i \overline{T_f} \rangle^i + \overline{\langle \mathbf{u}' \rangle^i \langle T_f' \rangle^i} + \langle \overline{{}^i \mathbf{u}'^i T_f'} \rangle^i)]\end{aligned} \tag{49}$$

The last three terms in (49) are additional unknowns coming from application of both processes of averaging, namely time and volume averaging. As mentioned above, they represent **dispersion** due to the *spatial deviations*, **turbulent heat flux** due to *time fluctuations* and **turbulent dispersion** due to both *time fluctuations* and *spatial deviations*. Models for thermal dispersion and for turbulent heat flux have been applied on separate to flows through clear and porous domains, respectively. To the best of the authors' knowledge, no work in the literature has proposed a general model encompassing all terms in (49).

Conclusions

In this work it was shown that, under the light of the double decomposition concept [8,9], both procedures employed to arrive at the macroscopic energy equation, namely *time-volume averaging* or *volume-time averaging*, lead to the same result for the case of incompressible flow in a rigid, homogeneous porous medium. The extra terms appearing in equation (49) still need to be modeled in terms of $\mathbf{u}_D$ and $\langle \overline{T_f} \rangle^i$. It is expected that further research on this subject be stimulated by the derivation herein.

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