Recent Mathematical Models for Turbulent Flow in Saturated Rigid Porous Media

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Turbulence models proposed for flow through permeable structures depend on the order of application of time and volume average operators. Two developed methodologies, following the two orders of integration, lead to different governing equations for the statistical quantities. The flow turbulence kinetic energy resulting in each case is different. This paper reviews recently published mathematical models developed for such flows. The concept of double decomposition is discussed and models are classified in terms of the order of application of time and volume averaging operators, among other peculiarities. A total of four major classes of models are identified and a general discussion on their main characteristics is carried out. Proposed equations for turbulence kinetic energy following time-space and space-time integration sequences are derived and similar terms are compared. Treatment of the drag coefficient and closure of the interfacial surface integrals are discussed. [DOI: 10.1115/1.1413243]

Introduction

On the basis of the pore Reynolds number, Re_p, the literature recognizes distinct flow regimes in porous media spanning from creeping flow ($\mathrm{Re}_p<1$) to fully turbulent regime ($\mathrm{Re}_p>300$). The mathematical treatment for high Reynolds flow has given rise to interesting discussions in the literature and remains a controversial issue.

For Re_p less than about 150, traditional analysis of flow in porous media (Darcy [1], Forchheimer [2], Brinkman [3], Ward [4], Whitaker [5], Bear [6], Vafai and Tien [7]) makes use of a Representative Elementary Volume (REV) for which transport equations are written. When the pore Reynolds number is greater than about 300, turbulence models in the literature follow two approaches. In the first one (Lee and Howell [8], Wang and Takle [9], Antohe and Lage [10], Getachew et al. [11]), governing equations for the mean and turbulent field are obtained by time-averaging the volume-averaged equations. We shall refer to those as A-L models. In the second methodology (Masuoka and Takatsu [12], Kuwahara et al. [13], Kuwahara and Nakayama [14], Takatsu and Masuoka [15], Nakayama and Kuwahara [16]), a volume-average operator is applied to the local time-averaged equation. Here, this second approach is named N-K model. A morphology-based closure has also been suggested (Travkin and Catton [17], Travkin et al. [18], Gratton et al. [19], Travkin and Catton [20], Travkin and Catton [21], Travkin et al. [22]) based on the Volume Average Theory. Use of such methodology, however, is regarded by many as of little practical use in engineering applications (Lage [23], p. 23). This third class of model will be here referred to as T-C approach. In the literature, all of these methodologies lead to different governing equations.

Motivated by the foregoing discussion, a preliminary proposal for a turbulence model for porous material was established (Pedras and de Lemos [24]). Then, a study on the different views in the literature has lead to the proposition of the *double-decomposition* idea (Pedras and de Lemos [25]) and to subsequent development of the earlier preliminary model (Pedras and de Lemos [26]). The double-decomposition concept led to a better characterization of the flow turbulent kinetic energy (Pedras and de Lemos [27]) and was a step before detailed numerical solution of the flow equations were carried out (Pedras and de Lemos [28]). Calculations were needed for adjusting the model and considered either the high Re k-ϵ closure (Rocamore and de Lemos [29]) as well as the Low Reynolds version of it (Pedras and de Lemos [28]). Full documentation of the modeling steps is detailed in de Lemos and Pedras [30], and Pedras and de Lemos [25].

Heat transfer analysis was also the subject of additional research (Rocamora and de Lemos [31,32]). One of the main motivations for this development was the ability to treat hybrid computational domains with a single mathematical tool. Hybrid systems have been calculated for the flow field (de Lemos and Pedras [33]), for nonisothermal recirculating flows in channels past a porous obstacle (Rocamora and de Lemos [34,35]) and through a porous insert (Rocamora and de Lemos [36,37]).

More specifically, in the work of Pedras and de Lemos [25,27], it was shown that the order of application of time and volume average operators was immaterial in regard to the final equations obtained for the mean flow. However, when obtaining a macroscopic transport equation for the turbulent kinetic energy, the order of application of averages will imply in a different quantity being transported. This is because there is an additional mathematical operation needed for forming the turbulent kinetic energy. This operation consists in the scalar product of the fluctuating velocity by its own transport equation. When this scalar product is taken after the volume integration process, as in A-L models (Lee and Howell [8], Wang and Takle [9], Antohe and Lage [10]), the quantity undergoing time integration is $\langle \mathbf{u}' \rangle^i \cdot \langle \mathbf{u}' \rangle^i$. Here, differently from the case of the mean flow equations, the two domains of integration are no longer independent of each other. On the other hand, when proceeding with the scalar product first at the microscopic level, with the N-K approach, a different variable is subjected to time integration $(\mathbf{u}' \cdot \mathbf{u}')$ (Masuoka and Takatsu [12], Kuwahara et al. [14], Kuwahara and Nakayama [13], Takatsu and Masuoka [37], Nakayama and Kuwahara [16]). In this second method, according to Pedras and de Lemos [27], a broader form of the turbulence kinetic energy is obtained and all microscopic effects are considered.

The objective of this paper is to classify and compare turbulence models for porous medium presented in the literature (de Lemos [38]). Proposed equations for the turbulent kinetic energy are rewritten in light of the double decomposition concept of Pedras and de Lemos [27] and the interrelationship between correspondent terms is discussed (de Lemos and Pedras [39]). Weakness and advantages of these two methodologies are discussed in

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an attempt to assess their practical use in engineering computation. It is expected that the contribution herein provide some insight to turbulence modelers devoted to analyze engineering systems and environmental flows which can be modeled as a porous structure having a fluid flowing in turbulent regime.

Macroscopic Fluctuating Velocity

The derivation to follow has been presented in de Lemos and Pedras [39]. To the best of the authors' knowledge, an equation for the volume-average velocity fluctuation was there derived, for the first time, in light of the double-decomposition concept of Pedras and de Lemos [40]. For clarity, some basic relationships from the double-decomposition idea are here included.

Double Decomposition. For a general fluid property, φ, the intrinsic and volumetric averages are related through the porosity ϕ as (Bear [6]),

$$\langle\varphi\rangle^i=\frac{1}{\Delta V_f}\int_{\Delta V_f}\varphi dV;\langle\varphi\rangle^v=\phi\langle\varphi\rangle^i;\phi=\frac{\Delta V_f}{\Delta V} \tag{1}$$

where ΔV_f is the volume of the fluid contained in ΔV. The property φ can then be defined as the sum of $\langle\varphi\rangle^i$ and its spatial variation within the REV, ${}^i\varphi$, as (Whittaker [5]),

$$\varphi=\langle\varphi\rangle^i+{}^i\varphi \tag{2}$$

Time fluctuations have to be considered when turbulence effects are of concern. The microscopic time-averaged equations are obtained from the instantaneous microscopic equations. For that, the time-average value of property, φ, associated with the fluid is given as:

$$\overline{\varphi}=\frac{1}{\Delta t}\int_t^{t+\Delta t}\varphi dt \tag{3}$$

where Δt is the integration time interval. The instantaneous property, φ, can be defined as the sum of the time average, $\overline{\varphi}$, plus the fluctuating component, φ':

$$\varphi=\overline{\varphi}+\varphi' \tag{4}$$

From the work of Pedras and de Lemos [27], and Rocamora and de Lemos [32], one can write for any flow property φ, combining decompositions (2) and (4),

$$\overline{\langle\varphi\rangle'}=\langle\overline{\varphi}\rangle^i \tag{5}$$

$$\langle\varphi\rangle^{i\prime}=\langle\varphi'\rangle^i \tag{6}$$

$${}^i\overline{\varphi}=\overline{{}^i\varphi} \tag{7}$$

Leading a full variable decomposition as (Pedras and de Lemos [40,41]):

$$\begin{aligned}\varphi&=\langle\overline{\varphi}\rangle^i+\langle\varphi'\rangle^i+{}^i\overline{\varphi}+{}^i\varphi'\\&=\overline{\langle\varphi\rangle^i}+\langle\varphi\rangle^{i\prime}+\overline{{}^i\varphi}+{}^i\varphi'\end{aligned} \tag{8}$$

Equation (8) comprises the *double decomposition* concept used in the development to follow. One should point out that (8) refers to any medium property over which the volume and time averaging operators are simultaneously applied. It is not restricted to fluid flow problems (e.g., compressible or incompressible, viscous or inviscid). Characterization of macroscopic fluctuation temperatures could well use the idea embodied in (8). The only limitation is the independence of *time* and *space* integration domains and, therefore, swelling, shrinking, or vibrating media are not considered within this frame work (see Pedras and de Lemos [25,27] for a discussion on the limitations of Eq. (8)). With these ideas in mind, an equation for macroscopic velocity fluctuations is shown next.

Macroscopic Velocity Fluctuation. The starting point for an equation for the flow turbulent kinetic energy is an equation for the microscopic velocity fluctuation $\mathbf{u}'$. Such a relationship can be written after subtracting the equation for the mean velocity $\overline{\mathbf{u}}$ from the instantaneous momentum equation, resulting in (Hinze [42], Warsi [43]):

$$\rho\left\{\frac{\partial\mathbf{u}'}{\partial t}+\nabla\cdot[\overline{\mathbf{u}}\mathbf{u}'+\mathbf{u}'\overline{\mathbf{u}}+\mathbf{u}'\mathbf{u}'-\overline{\mathbf{u}'\mathbf{u}'}]\right\}=-\nabla p'+\mu\nabla^2\mathbf{u}'. \tag{9}$$

Now, the volumetric average of (9) will give,

$$\begin{aligned}&\rho\frac{\partial}{\partial t}(\phi\langle\mathbf{u}'\rangle^i)+\rho\nabla+\{\phi[\langle\overline{\mathbf{u}}\mathbf{u}'\rangle^i+\langle\mathbf{u}'\overline{\mathbf{u}}\rangle^i+\langle\mathbf{u}'\mathbf{u}'\rangle^i-\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i]\}\\&\quad=-\nabla(\phi\langle p'\rangle^i)+\mu\nabla^2(\phi\langle\mathbf{u}'\rangle^i)+\mathbf{R}'\end{aligned} \tag{10}$$

where,

$$\mathbf{R}'=\frac{\mu}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\nabla\mathbf{u}')dS-\frac{1}{\Delta V}\int_{A_i}\mathbf{n}p'dS \tag{11}$$

is the fluctuating part of the total drag due to the porous structure.

Expanding further the divergent operator in (10) in light of (8), one ends up with an equation for $\langle\mathbf{u}'\rangle^i$ as,

$$\begin{aligned}&\rho\frac{\partial}{\partial t}(\phi\langle\mathbf{u}'\rangle^i)+\rho\nabla\cdot\{\phi[\langle\overline{\mathbf{u}}\rangle^i\langle\mathbf{u}'\rangle^i+\langle\mathbf{u}'\rangle^i\langle\overline{\mathbf{u}}\rangle^i+\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i\\&\quad+\langle{}^i\overline{\mathbf{u}}{}^i\mathbf{u}'\rangle^i+\langle{}^i\mathbf{u}'{}^i\overline{\mathbf{u}}\rangle^i+\langle{}^i\mathbf{u}'{}^i\mathbf{u}'\rangle^i-\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}-\overline{\langle{}^i\mathbf{u}'{}^i\mathbf{u}'\rangle^i}]\}\\&\quad=-\nabla(\phi\langle p'\rangle^i)+\mu\nabla^2(\phi\langle\mathbf{u}'\rangle^i)+\mathbf{R}'\end{aligned} \tag{12}$$

Macroscopic Turbulent Kinetic Energy

The objective of this section is to derive both transport equations for k_m and $\langle k\rangle^i$ in order to compare similar terms.

Equation for $k_m=\overline{\langle\mathbf{u}'\rangle^i\cdot\langle\mathbf{u}'\rangle^i}/2$. From the instantaneous microscopic continuity equation for a constant property fluid one has,

$$\nabla\cdot(\phi\langle\mathbf{u}\rangle^i)=0\Rightarrow\nabla\cdot[\phi(\langle\overline{\mathbf{u}}\rangle^i+\langle\overline{\mathbf{u}}'\rangle^i)]=0 \tag{13}$$

with time average,

$$\nabla\cdot(\phi\langle\overline{\mathbf{u}}\rangle^i)=0 \tag{14}$$

From (13) and (14) one verifies that,

$$\nabla\cdot(\phi\langle\mathbf{u}'\rangle^i)=0 \tag{15}$$

Taking the scalar product of (10) by $\langle\mathbf{u}'\rangle^i$, making use of (13)-(14)-(15) and time averaging it, an equation for k_m will have for each of its terms (note that ϕ is here considered as independent of time):

$$\overline{\rho\langle\mathbf{u}'\rangle^i\cdot\frac{\partial}{\partial t}(\phi\langle\mathbf{u}'\rangle^i)}=\rho\frac{\partial(\phi k_m)}{\partial t} \tag{16}$$

$$\begin{aligned}\overline{\rho\langle\mathbf{u}'\rangle^i\cdot\{\nabla\cdot(\phi\langle\overline{\mathbf{u}}\mathbf{u}'\rangle^i)\}}&=\overline{\rho\langle\mathbf{u}'\rangle^i\cdot\{\nabla\cdot[\phi\langle\overline{\mathbf{u}}\rangle^i\langle\mathbf{u}'\rangle^i+\phi\langle{}^i\overline{\mathbf{u}}{}^i\mathbf{u}'\rangle^i]\}}\\&=\rho\nabla\cdot[\phi\langle\overline{\mathbf{u}}\rangle^ik_m]\\&\quad+\rho\overline{\langle\mathbf{u}'\rangle^i\cdot\{\nabla\cdot[\phi\langle{}^i\overline{\mathbf{u}}{}^i\mathbf{u}'\rangle^i]\}}\end{aligned} \tag{17}$$

$$\begin{aligned}\overline{\rho\langle\mathbf{u}'\rangle^i\cdot\{\nabla\cdot(\phi\langle\mathbf{u}'\overline{\mathbf{u}}\rangle^i)\}}&=\overline{\rho\langle\mathbf{u}'\rangle^i\{\nabla\cdot[\phi\langle\mathbf{u}'\rangle^i\langle\overline{\mathbf{u}}\rangle^i+\phi\langle{}^i\mathbf{u}'{}^i\overline{\mathbf{u}}\rangle^i]\}}\\&=\rho\phi\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}:\nabla\langle\overline{\mathbf{u}}\rangle^i\\&\quad+\rho\overline{\langle\mathbf{u}'\rangle^i\cdot\{\nabla\cdot[\phi\langle{}^i\mathbf{u}'{}^i\overline{\mathbf{u}}\rangle^i]\}}\end{aligned} \tag{18}$$

$$\rho\overline{\langle\mathbf{u}'\rangle^i\cdot\{\nabla\cdot(\phi\langle\mathbf{u}'\mathbf{u}'\rangle^i)\}}$$
$$=\rho\overline{\langle\mathbf{u}'\rangle^i\cdot\{\nabla\cdot[\phi\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i+\phi\langle^i\mathbf{u}'^i\mathbf{u}'\rangle^i]\}}$$
$$=\rho\nabla\cdot\left[\phi\langle\mathbf{u}'\rangle^i\frac{\langle\mathbf{u}'\rangle^i\cdot\langle\mathbf{u}'\rangle^i}{2}\right]+\rho\overline{\langle\mathbf{u}'\rangle^i\cdot\{\nabla\cdot[\phi\langle^i\mathbf{u}'^i\mathbf{u}'\rangle^i]\}} \quad (19)$$

$$\rho\overline{\langle\mathbf{u}'\rangle^i\cdot\{\nabla\cdot(-\phi\overline{\langle\mathbf{u}'\mathbf{u}'\rangle^i})\}}=0 \quad (20)$$

$$-\overline{\langle\mathbf{u}'\rangle^i\cdot\nabla(\phi\langle p'\rangle^i)}=-\nabla\cdot\overline{[\phi\langle\mathbf{u}'\rangle^i\langle p'\rangle^i]} \quad (21)$$

$$\mu\overline{\langle\mathbf{u}'\rangle^i\cdot\nabla^2(\phi\langle\mathbf{u}'\rangle^i)}=\mu\nabla^2(\phi k_m)-\rho\phi\epsilon_m \quad (22)$$

$$\overline{\langle\mathbf{u}'\rangle^i\cdot\mathbf{R}'}\equiv 0 \quad (23)$$

where $\epsilon_m=\nu\overline{\nabla\langle\mathbf{u}'\rangle^i:(\nabla\langle\mathbf{u}'\rangle^i)^T}$. In handling (21) the porosity ϕ was assumed to be constant only for simplifying the manipulation to be shown next. This procedure, however, does not represent a limitation in deriving a general form transport equation for k_m since term (21) will require further modeling.

Another important point is the treatment given to the scalar product shown in (23). Here, a different view from the work in the Lee and Howell [8], Wang and Takle [9], Antohe and Lage [10], and Getachewa et al. [11], is considered. The fluctuating drag form $\mathbf{R}'$ acts through the solid-fluid interfacial area and, as such, on fluid particles at rest. The fluctuating mechanical energy represented by the operation in (23) is not associated with any fluid particle movement and, as a result, is here considered to be of null value. This point shall be further discussed later in this paper.

A final equation for k_m gives,

$$\rho\frac{\partial(\phi k_m)}{\partial t}+\rho\nabla\cdot[\phi\langle\overline{\mathbf{u}}\rangle^i k_m]$$
$$=-\rho\nabla\cdot\left\{\overline{\phi\langle\mathbf{u}'\rangle^i\left[\frac{\langle p'\rangle^i}{\rho}+\frac{\langle\mathbf{u}'\rangle^i\cdot\langle\mathbf{u}'\rangle^i}{2}\right]}\right\}$$
$$+\mu\nabla^2(\phi k_m)-\rho\phi\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}:\nabla\langle\overline{\mathbf{u}}\rangle^i-\rho\phi\epsilon_m-D_m \quad (24)$$

where

$$D_m=\rho\overline{\langle\mathbf{u}'\rangle^i\cdot\{\nabla\cdot[\phi(\langle^i\overline{\mathbf{u}}^i\mathbf{u}'\rangle^i+\langle^i\mathbf{u}'^i\overline{\mathbf{u}}\rangle^i+\langle^i\mathbf{u}'^i\mathbf{u}'\rangle^i)]\}} \quad (25)$$

represents the dispersion of k_m given by the last term on the right of (17), (18), and (19), respectively. Interesting to point out is that this term can be both of negative or positive sign.

The first term on the right of (24) represents the turbulent diffusion of k_m and is normally modeled via a diffusion-like expression resulting for the transport equation (Antohe and Lage [10], Getachewa et al. [11]),

$$\rho\frac{\partial(\phi k_m)}{\partial t}+\rho\nabla\cdot[\phi\langle\overline{\mathbf{u}}\rangle^i k_m]$$
$$=\nabla\cdot\left[\mu+\frac{\mu_{t_m}}{\sigma_{k_m}}\nabla(\phi k_m)\right]+P_m-\rho\phi\epsilon_m-D_m \quad (26)$$

where

$$P_m=-\rho\phi\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}:\nabla\langle\overline{\mathbf{u}}\rangle^i \quad (27)$$

is the production rate of k_m due to the gradients of the macroscopic time-mean velocity $\langle\overline{\mathbf{u}}\rangle^i$.

Lee and Howell [8], Wang and Takle [9], Antohe and Lage [10], and Getachewa et al. [11], made use of the above equation for k_m considering for $\mathbf{R}'$ (11) the Darcy-Forchheimer extended model with macroscopic time-fluctuation velocities $\langle\mathbf{u}'\rangle^i$. They have also neglected all dispersion terms that were here grouped into D_m (25). Note also that the order of application of both volume- and time-average operators in this case cannot be changed. The quantity k_m is defined by applying first the volume operator to the fluctuating velocity field.

Equation for $\langle k\rangle^i=\langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2$. The other procedure for composing the flow turbulent kinetic energy is to take the scalar product of (9) by the microscopic fluctuating velocity $\mathbf{u}'$. Then applying both time and volume-operators for obtaining an equation for $\langle k\rangle^i=\langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2$. It is worth noting that in this case the order of application of both operations is immaterial since no additional mathematical operation (the scalar product) is conducted in between the averaging processes. Therefore, this is the same as applying the volume operator to an equation for the microscopic k.

The volumetric average of a transport equation for k has been carried out in detail by de Lemos and Pedras [30], and Pedras and de Lemos [40], and for that only the final resulting equation is here presented. It reads,

$$\rho\left[\frac{\partial}{\partial t}(\phi\langle k\rangle^i)+\nabla\cdot(\overline{\mathbf{u}}_D\langle k\rangle^i)\right]$$
$$=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_k}\right)\nabla(\phi\langle k\rangle^i)\right]+P_i+G_i-\rho\phi\langle\epsilon\rangle^i \quad (28)$$

where

$$P_i=-\rho\overline{\langle\mathbf{u}'\mathbf{u}'\rangle^i}:\nabla\overline{\mathbf{u}}_D \quad (29)$$

$$G_i=c_k\rho\phi\frac{\langle k\rangle^i|\overline{\mathbf{u}}_D|}{\sqrt{K}} \quad (30)$$

are the production rate of $\langle k\rangle^i$ due to mean gradients of the seepage velocity and the generation rate of intrinsic k due to the presence of the porous matrix. As mentioned, Eq. (28) has been proposed by Pedras and de Lemos [40], where more details on its derivation can be found. The constant c_k was numerically determined in Pedras and de Lemos [41,44] for different media and for a wide range of porosity and Reynolds numbers. In spite of having distinct cases, a unique value of 0.28 was found to be suitable for most calculations.

Comparison of Macroscopic Transport Equations. A comparison between terms in the transport equation for k_m and $\langle k\rangle^i$ can now be conducted. Pedras and de Lemos [27] have already shown the connection between these two quantities as being,

$$\langle k\rangle^i=\langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2=\overline{\langle\mathbf{u}'\rangle^i\cdot\langle\mathbf{u}'\rangle^i}/2+\langle\overline{^i\mathbf{u}'\cdot{}^i\mathbf{u}'}\rangle^i/2=k_m+\langle\overline{^i\mathbf{u}'\cdot{}^i\mathbf{u}'}\rangle^i/2 \quad (31)$$

Expanding the correlation forming the production term P_i by means of (2), a connection between the two generation rates can also be written as,

$$P_i=-\rho\overline{\langle\mathbf{u}'\mathbf{u}'\rangle^i}:\nabla\overline{\mathbf{u}}_D=-\rho(\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}:\nabla\overline{\mathbf{u}}_D+\overline{\langle^i\mathbf{u}'^i\mathbf{u}'\rangle^i}:\nabla\overline{\mathbf{u}}_D)$$
$$=P_m-\rho\overline{\langle^i\mathbf{u}'^i\mathbf{u}'\rangle^i}:\nabla\overline{\mathbf{u}}_D \quad (32)$$

One can note that all production rates of k_m due to the mean flow constitutes only part of the general production rate responsible for maintaining the overall level of $\langle k\rangle^i$.

The dissipation rates also carry a correspondence if one expands

$$\langle\epsilon\rangle^i=\nu\overline{\langle\nabla\mathbf{u}':(\nabla\mathbf{u}')^T\rangle^i}$$
$$=\nu\overline{\langle\nabla\mathbf{u}'\rangle^i:[\langle\nabla\mathbf{u}'\rangle^i]^T}+\nu\overline{\langle^i(\nabla\mathbf{u}'):{}^i(\nabla\mathbf{u}')^T\rangle^i}$$
$$=\frac{\nu}{\phi^2}\overline{\nabla(\phi\langle\mathbf{u}'\rangle^i):[\nabla(\phi\langle\mathbf{u}'\rangle^i)]^T}+\nu\overline{\langle^i(\nabla\mathbf{u}'):{}^i(\nabla\mathbf{u}')^T\rangle^i} \quad (33)$$

Considering further constant porosity,

Table 1 Classification of turbulence models for porous media.

Model Class	Authors	General characteristics and treatment of surface integrals	Sequence of integration	Applications
A-L	Lee & Howell, 1987, Wang & Takle, 1995, Antohe & Lage, 1997, Getachewa *et al.*, 2000.	Surface integrals are not applied since models are based on macroscopic quantities subjected to time-averaging only.	Space-time	Only theory presented. Numerical results using this model are found in Chan et al., 2000.
N-K	Masuoka & Takatsu, 1996, Kuwahara *et al,* 1998, Kuwahara & Nakayama, 1998, Takatsu & Masuoka, 1998, Nakayama & Kuwahara, 1999.	Masuoka & Takatsu, 1996, assumed a non-null value in their eqn. (11) for the turbulent shear stress $\mathbf{S}_t = -\rho\overline{\mathbf{u}'\mathbf{u}'}$ along the interfacial area A_i. Takatsu & Masuoka, 1998, assume for their volume integral in eqn. (14) a different form zero value for $\mathbf{d} = (\mu/\rho + \mu_t/\sigma_k\rho)\nabla k$ at the interface A_i.	Time-space	Microscopic computations on periodic cell of square rods. Macroscopic model computations presented.
T-C	Gratton *et al.,* 1994, Travkin & Catton, 1992, 1993, 1995, 1998,Travkin *et al.,* 1993, Travkin *et al.,* 1999	Morphology-based theory. Surface integrals and volume-average operators depend on media morphology.	Time-space	Only theory, no closure for the macroscopic equation is presently available.
P-dL	Pedras & de Lemos, 2000a, 2001a, 2000b, Rocamora & de Lemos, 2000a.	Double-decomposition theory. Surface integrals involving null quantities at surfaces are neglected. The connection between ***space-time*** and ***time-space*** theories is unveiled.	Time-space	Microscopic computation on periodic cell of circular (Pedras & de Lemos, 2001b) and elliptic Pedras & de Lemos, 2001c, 200d) rods. Macroscopic computations for porous media presented. Results for hybrid domains are found in de Lemos & Pedras, 2000b and Rocamora & de Lemos, 2000b, 2000c, 2000d.

$$\langle \epsilon \rangle^i = \epsilon_m + \nu\overline{\langle^i(\nabla \mathbf{u}'):^i(\nabla \mathbf{u}')^T\rangle^i} \tag{34}$$

Equation (34) indicates that an additional dissipation rate is necessary to fully account for the energy decay process inside the R.E.V. It is worth noting that (31), (32), and (34) seems to suggest that models considering $\langle k \rangle^i$ are by far more complete than theories based on k_m.

General Classification of Turbulence Models for Porous Media

Based on the derivations above, one can establish a general classification of the models presented so far in the literature (de Lemos [38]). Table 1 classifies all proposals into four major categories. These classes are based on the sequence of application of averaging operators, on the handling of surface integrals and on the application reported so far.

The A-L models make use of transport equations for $k_m = \overline{\langle \mathbf{u}' \rangle^i \cdot \langle \mathbf{u}' \rangle^i}/2$ instead of $\langle k \rangle^i = \overline{\langle \mathbf{u}' \cdot \mathbf{u}' \rangle^i}/2$. Consequently, this methodology applies only time-averaging procedure to already established macroscopic equations (see for example Hsu and Cheng [45], for macroscopic equations). In this sense, the sequence *space-time* integration is employed and surface integrals are not manipulated since macroscopic quantities are the sole independent variables used. Application of this theory is found in Chan et al. [46].

N-K models constitute the second class of models here compiled. It is interesting to mention that Masuoka and Takatsu [12], assumed a non-null value for the turbulent shear stress, $\mathbf{S}_t = -\rho\overline{\mathbf{u}'\mathbf{u}'}$, along the interfacial area A_i in their Eq. (11). With that, their surface integral $\int_{A_i} \mathbf{S}_t \cdot \mathbf{n} dA$ was associated with the Darcy flow resistance term. Yet, using the Boussinesq approximation as in their Eq. (7), $\mathbf{S}_t = 2\mu_t \mathbf{D} - (2/3)k\mathbf{I}$, one can also see that both μ_t and k will vanish at the surface A_i, ultimately indicating that the surface integral in question is actually equal to zero. Similarly, Takatsu and Masuoka [15], assumed for their surface integral in Eq. (14), $\int_{A_i} \mathbf{d} \cdot \mathbf{n} dA$, a non-null value where $\mathbf{d} = (\mu/\rho + \mu_t/\sigma_k\rho)\nabla k$. Here also it is worth noting that $\nabla k = \overline{\mathbf{u}' \cdot (\nabla \mathbf{u}')^T}$ and that, at the interface A_i, $\nabla k = 0$ due to the non-slip condition. Consequently, also in this case the surface integral of $\mathbf{d}$ over A_i is of zero value. In regard to the average operators used, N-K models follow the *time-space* integration sequence. Calibration of the model required microscopic computations on a period cell of square rods. Macroscopic results in a channel filled with a porous material was also a test case run by Nakayama and Kuwahara [16].

The work developed in a series of papers using a morphology-oriented theory is here group in the T-C model category shown in Table 1. In this morphology-based theory, surface integrals resulting after application of volume-average operators depend on the media morphology. Governing equations set up for turbulent flow, although complicated at first sight, just follow usual volume integration technique applied to standard k-ϵ and k-L turbulence models. In this sense, time-space integration sequence is followed. No closure is proposed for the unknown surface integrals (and morphology parameters) so that practical applications of such development in solving *real-world* engineering flows is still a challenge to be overcome. Nevertheless, the developed theory seems to be mathematically correct even though additional *ad-hoc* information is still necessary to fully model the remaining unknowns and medium-dependent parameters.

Lastly, the model group named P-dL uses the recently developed *double-decomposition* theory just reviewed above. In this development, all surface integrals involving null quantities at interface A_i are neglected. The connection between space-time and time-space theories is made possible due to the splitting of the dependent variables into four (rather than two) components, as expressed by Eq. (8). For the momentum and energy equations, the double-decomposition approach has proven that either time-space or space-time order of application of averaging operators is immaterial. For the turbulence kinetic energy equation, however, the order of application of such mathematical operators will define different quantities being transported (Pedras and de Lemos [27], Rocamora and de Lemos [32]). Microscopic computation on a periodic cell of circular (Pedras and de Lemos [41]) and elliptic (Pedras and de Lemos [47]) rods were used in order to calibrate the proposed model. Pedras and de Lemos [40] further presented macroscopic computations for flow in a channel filled with a porous material. Further results for hybrid domains (*porous medium-clear fluid*) are found in de Lemos and Pedras [33] and Rocamora and de Lemos [34–36]).

Discussion and Conclusions

This paper presented the two views in the literature for characterizing the turbulence kinetic energy for flow in porous media. The two transport equations where derived in light of the double

decomposition concept and a comparison between the production and dissipation terms are presented. A general classification of all models published so far was established. The discussion below further compares the two views in the literature.

In the path followed by Lee and Howell [8], Wang and Takle [9] and Antohe and Lage [10], the drag term **R** was represented by a Darcy-Forchheimer extended model in its usual form,

$$\mathbf{R}=-\left[\frac{\mu\phi}{K}\mathbf{u}_D+\frac{c_F\phi\rho|\mathbf{u}_D|\mathbf{u}_D}{\sqrt{K}}\right] \tag{35}$$

where c_F is a constant. Time decomposition was then applied to the Darcy velocity in (35) and the 4th, 5th, 6th, and 8th terms in the divergent of (12) were neglected. A few comments on this methodology for setting up a transport equation for the flow kinetic energy seems timely.

The Darcy-Forchheimer extended model is based on average values obtained after comparing bulk flow rates to bulk pressure drops across beds of saturated media. It compares, accordingly, time-averaged quantities although a fluctuating component for the macroscopic velocity is defined and has been used throughout the development shown above. Applying then a time-decomposition operation to (35) in order to represent extra turbulent kinetic energy for the flow in a porous matrix seems to neglect the fact that such terms were proposed based on ''time-independent'' quantities. Accordingly, Eq. (35) seems to be a model for the *time-mean* drag rather than for the *instantaneous* force (11).

Also, one interesting point in the development of an equation for k_m is the treatment given to the scalar product (23). This term represents the statistical value of the fluctuating mechanical energy associated with the fluctuating drag $\mathbf{R}'$ given by (11). The fluctuating drag $\mathbf{R}'$, although different from zero, acts through the solid-fluid interface A_i and, as such, on fluid particles ''at rest.'' As a consequence, this force should not contribute to producing mechanical energy within the flow.

Accordingly, the work done by a force acting on a particle moving along a certain distance is the scalar product of this force, **F**, by the distance, $d\mathbf{r}$, such as

$$dW=\mathbf{F}\cdot d\mathbf{r} \tag{36}$$

The work rate or power is then defined as,

$$\dot{W}=\frac{dW}{dt}=\mathbf{F}\cdot\frac{d\mathbf{r}}{dt}=\mathbf{F}\cdot\mathbf{u} \tag{37}$$

or say, in Eq. (37) **u** is the velocity of the particle on which the force **F** is being applied.

Following Bird et al. [48], the microscopic (local and instantaneous) mechanical energy equation is obtained starting with the momentum Cauchy equation,

$$\rho\frac{D\mathbf{u}}{Dt}=\nabla\cdot\mathbf{T}+\rho\mathbf{g} \tag{38}$$

where **T** is the stress tensor. All terms in (38) represent forces per unit volume acting on a fluid element having velocity **u**. Following the concept embodied in (37), the mechanical energy produced by each one of these forces is given by the scalar product of (38) and the local velocity **u**,

$$\rho\frac{D}{Dt}\left(\frac{1}{2}u^2\right)=\mathbf{u}\cdot\nabla\cdot\mathbf{T}+\mathbf{u}\cdot\rho\mathbf{g} \tag{39}$$

It is clear to see that all terms in (39) vanish due to the non-slip condition at a solid stagnant wall. Text books in Fluid Mechanics (e.g., Bird et al. [48], Fox and McDonald [49]) comment that in the neighborhood of a fluid particle, stagnant solid walls cannot promote mechanical energy. Or say, solid walls can only generate mechanical energy within the flow if the wall itself is moving (e.g., a rotating turbine blade). By means of (37), this mechanical energy would be given by the scalar product of the force exerted by the wall on the fluid and the fluid particle velocity (or the velocity of the fluid particle in contact to the moving wall).

It is also clear that boundary forces modify the fluid pressure field, which, in turn, modifies fluid velocity and then the mechanical energy within the flow. However, the momentum equation in its differential form considers only the forces acting in the vicinity of a moving particle. At walls, there is no fluid movement except in the case of moving surfaces. Following this line of thought, no term derived from (35) could then contribute to either increase or decrease the overall value of the flow turbulent kinetic energy. An exception would be a moving (vibrating) porous structure and, in this case, energy would be added to the fluid. It is recognized, however, that additional terms in the *k*-equation due to the solid structure are necessary (Nakayama and Kuwahara [16]). Different proposals are made in the literature and recent work in this area is expected to improve current model assumptions (Pedras and de Lemos [27]).

Also, for a homogeneous, fully-developed unidimensional flow through a porous bed, all terms appearing in the k_m-equation and originated from the time decomposition of Darcy-Forchheimer extended model will be negative (note that all drag forces in (35) are ''negative'' and will lead to ''sink'' terms in the k_m-equation). As a consequence, the only possible solution for this case will be $k_m=0$. This, in fact, was the conclusion reached by Antohe and Lage [10]. However, Nakayama and Kuwahara [16], points out that for this same situation a certain level of turbulent kinetic energy must stay as long as the presence of porous matrix keeps generating it. This disagreement could be explained based on the fact that each work talks about a different quantity (k_m and $\langle k\rangle^i$). For flow in such infinite medium, the only generating mechanism is given by term G_i in (28) causing a non-null value for $\langle k\rangle^i$. This would be equivalent to considering the decay of turbulence behind a grid and analyzing the porous structure as a sequence of closely packed grids. Thus, mechanical energy continuously extracted from the mean flow, by gradients of *microscopic* velocity, feeds the macroscopic turbulence field. This mechanism is modeled by the generating term G_i. On the other hand, the production *P*-terms in (32), for both *k* forms, will be zero due to null gradients of the *macroscopic* mean velocity.

Applying this same reasoning to the macroscopic momentum equation including body forces (Hsu and Cheng [45]),

$$\rho\left[\frac{\partial}{\partial t}(\phi\langle\mathbf{u}\rangle^i)+\nabla\cdot(\phi\langle\mathbf{uu}\rangle^i)\right]=-\nabla(\phi\langle p\rangle^i)+\mu\nabla^2(\phi\langle\mathbf{u}\rangle^i)+\phi\rho\mathbf{g}+\mathbf{R} \tag{40}$$

where

$$\mathbf{R}=\frac{\mu}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\nabla\mathbf{u})dS-\frac{1}{\Delta V}\int_{A_i}\mathbf{n}p\,dS \tag{41}$$

one verifies that the five first terms represent forces acting on fluid particles whose macroscopic mean velocity is given by $\langle\mathbf{u}\rangle^\nu$ or $\phi\langle\mathbf{u}\rangle^i$. The last term, **R**, acts on particles that are located on the interfacial area A_i, (**R** is a surface force divided by volume). If the interfacial area A_i, moves (vibrates) then the mechanical energy produced will be the scalar product of **R** and the velocity of A_i, otherwise **R** will not produce mechanical energy.

In several papers, (Vafai and Tien [7], Hsu and Cheng [45], Antohe and Lage [10], among others) **R** is modeled as a function of the Darcy velocity, $\langle\mathbf{u}\rangle^\nu$. However, that does not mean that **R** acts on particles having an average velocity $\langle\mathbf{u}\rangle^\nu$. Consider, for example, *creeping flow* around a sphere where both viscous drag, F_μ, and form drag, F_p, are obtained by integrating viscous and pressure forces, respectively, over the sphere surface. The total drag is given by,

$$F_p+F_\mu=6\pi\mu R u_\infty \tag{42}$$

where R is the radius of the sphere and u_∞ is the free stream

velocity. In spite of having the total drag described as a function of u_∞, this force acts on stagnant particles and, as such, can not produce mechanical energy within the flow. Likewise, forces $\mathbf{R}'$ (11) and $\mathbf{R}$ (41) that appear in the governing equations for porous media cannot produce mechanical energy unless the porous structure itself is allowed to move or vibrate. In this case, mechanical energy is added to the fluid via the action of the porous structure.

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