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Macroscopic turbulence modeling for incompressible flow through undeformable porous media

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Abstract

The literature presents two different methodologies for developing turbulent models for flow in a porous medium. The first one starts with the macroscopic equations using the extended Darcy–Forchheimer model. The second method makes use, first, of the Reynolds-averaged equations. These two methodologies lead to distinct set of equations for the k–ε model. The present work details a mathematical model for turbulent flow in porous media following the second path, or say, space-integrating the equations for turbulent flow in clear fluid. In order to account for the porous structure, an additional term is included in the sources for k and ε. A methodology is followed for determining the additional constant proposed. The equations for the microscopic flow were numerically solved inside a periodic elementary cell. The porous structure was approximated by an infinite array of circular rods. The method SIMPLE and a non-orthogonal boundary-fitted coordinate system were employed. Integrated parameters where compared to the existing data for fully developed homogeneous flow through porous media. Preliminary results are in agreement with numerical experiments presented in the literature. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Engineering systems based on fluidized bed combustion, enhanced oil reservoir recovery, combustion in an inert porous matrix, underground spreading of chemical waste and chemical catalytic reactors are just a few examples of applications of the study of flow through porous media. Due to its broad range of applications in science and industry this interdisciplinary field has gained extensive attention lately. In a broader sense, the study of porous media embraces fluid and thermal sciences, materials, chemical, geothermal, petroleum and combustion engineering.

Based on the so-called pore Reynolds number Re_p the literature recognizes distinct flow regimes, namely: (a) Darcy or creeping flow regime ($Re_p < 1$); (b) Forchheimer flow regime ($1 \sim 10 < Re_p < 150$); (c) post-Forchheimer flow regime (unsteady laminar flow, $150 < Re_p < 300$); (d) fully turbulent flow ($Re_p > 300$). The mathematical description of the last regime has given rise to interesting discussions in the literature and remains a controversial issue.

For $Re_p < 150$, classical mathematical treatment of flow in porous media [1–7] invokes the notion of a representative elementary volume (REV) for which balance equations governing momentum, energy and mass transfer are written. Models based on this macroscopic (*volume-averaged*) point of view lose details on the flow pattern inside the REV and, together with ad-hoc information, give results on global flow characteristics.

For high Reynolds number ($Re_p > 300$), however, turbulence models presented in the literature follow two different approaches. In the first one [8–10], governing equations for the mean and turbulent fields are obtained by time-averaging and the volume-averaged equations. In the second method [11–16], a volume-average operator is applied to the local time-averaged equation. Or

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Nomenclature

c_k	constant in the extra production term for k-equation
D	rod diameter
H	height of periodic cell
k	turbulence kinetic energy (TKE), $k = \overline{\mathbf{u}' \cdot \mathbf{u}'}/2$
$\langle k \rangle^i$	intrinsic (fluid) average for k, $\langle k \rangle^i = \langle \overline{\mathbf{u}' \cdot \mathbf{u}'} \rangle^i /2$
k_m	TKE based on the fluctuation of $\langle \bar{\mathbf{u}} \rangle^i$, $k_m = \overline{\langle \mathbf{u}' \rangle^i \cdot \langle \mathbf{u}' \rangle^i}/2$
k_ϕ	fully developed value of $\langle k \rangle^i$
K	medium permeability
p	pressure
Re_p	pore Reynolds number
Re_H	Reynolds number based on H
S	length of periodic cell, $S = 2H$
$\mathbf{u}$	microscopic velocity vector
$\bar{\mathbf{u}}_\mathrm{D}$	Darcy velocity vector
$\langle \bar{\mathbf{u}} \rangle^i$	intrinsic velocity vector

Greek symbols

ε	dissipation rate of k, $\varepsilon = \mu \overline{\nabla \mathbf{u}' : (\nabla \mathbf{u}')^\mathrm{T}}/\rho$
$\langle \varepsilon \rangle^i$	intrinsic (fluid) average for ε
ε_ϕ	fully developed value of $\langle \varepsilon \rangle^i$
ΔV	representative elementary volume (REV)
ΔV_f	volume of fluid inside ΔV
φ	general variable
ϕ	porosity
$\langle \varphi \rangle^i$	intrinsic (fluid) average of φ
${}^i\varphi$	spatial deviation from intrinsic average of φ
$\mu_{t\phi}$	macroscopic coefficient of exchange for porous media
μ	fluid viscosity
σ_k	turbulent Prandtl number for $\langle k \rangle^i$
σ_ε	turbulent Prandtl number for $\langle \varepsilon \rangle^i$
ρ	fluid density

say, in the first case, volume-average is taken first and then time averaging is applied. In the latter method, the order of averaging is reversed. In the literature, these two different approaches lead to different governing equations and, ultimately, to contradicting overall conclusions.

Motivated by the foregoing controversy, the objective of this work is to present a two-equation turbulence model for flow through a rigid saturated medium. It is shown that for the macroscopic momentum equation the order of integration is immaterial in regard to the final expression obtained. Also, the turbulence kinetic energy (TKE) resulting from application of the two-averaging operators, following both orders of integration, are different. The connection between these two quantities is here discussed upon. Calibration of the proposed model includes solution of the flow governing equation within a periodic computational cell. Comparisons with similar numerical results in the literature give support to the modeling ideas herein.

2. The averaging operators

2.1. Local volume-average

The macroscopic governing equation for flow through a porous substratum can be obtained by volume averaging the corresponding microscopic equations over a REV, ΔV [6, Fig. 1]. For a general fluid property, the intrinsic and volumetric averages are related through the porosity ϕ as

$$\langle \varphi \rangle^i = \frac{1}{\Delta V_\mathrm{f}} \int_{\Delta V_\mathrm{f}} \varphi \, \mathrm{d}V, \quad \langle \varphi \rangle^v = \phi \langle \varphi \rangle^i, \quad \phi = \frac{\Delta V_\mathrm{f}}{\Delta V}, \qquad (1)$$

where δV_f is the volume of the fluid contained in ΔV. The property φ can then be defined as the sum of $\langle \varphi \rangle^i$ and a term related to its spatial variation within the REV, ${}^i\phi$, as [5],

$$\varphi = \langle \varphi \rangle^i + {}^i\varphi. \qquad (2)$$

From (1) and (2), one derives $\langle {}^i\varphi \rangle^i = 0$. Fig. 1 illustrates the idea underlined by Eq. (2) for the value of a property of vectorial nature (e.g., velocity) in a position $\mathbf{x}$. The spatial deviation is the difference between the real value (microscopic) and its intrinsic (fluid-based average) value.

For deriving the flow governing equations, it is necessary to know the relationship between the volumetric average of derivatives and the derivatives of the volumetric average. These relationships are presented in a number of works, namely Whitaker [5], Gray and Lee [17], Slattery, 1967 [18], and others, being known as the theorem of local volumetric average. They are written as:

$$\langle \nabla \varphi \rangle^v = \nabla \big(\phi \langle \varphi \rangle^i \big) + \frac{1}{\Delta V} \int_{A_i} \mathbf{n} \varphi \, \mathrm{d}S, \qquad (3)$$

$$\langle \nabla \cdot \varphi \rangle^v = \nabla \cdot \big(\phi \langle \varphi \rangle^i \big) + \frac{1}{\Delta V} \int_{A_i} \mathbf{n} \cdot \varphi \, \mathrm{d}S \qquad (4)$$

and

$$\left\langle \frac{\partial \varphi}{\partial t} \right\rangle^v = \frac{\partial}{\partial t} \big(\phi \langle \varphi \rangle^i \big) - \frac{1}{\Delta V} \int_{A_i} \mathbf{n} \cdot (\mathbf{u}_i \varphi) \, \mathrm{d}S, \qquad (5)$$

where A_i and $\mathbf{u}_i$ are the interfacial area and velocity of phase f and $\mathbf{n}$ is the unity vector normal to A_i.The area A_i should not be confused with the surface area surrounding volume ΔV in Fig. 1. To the interested reader

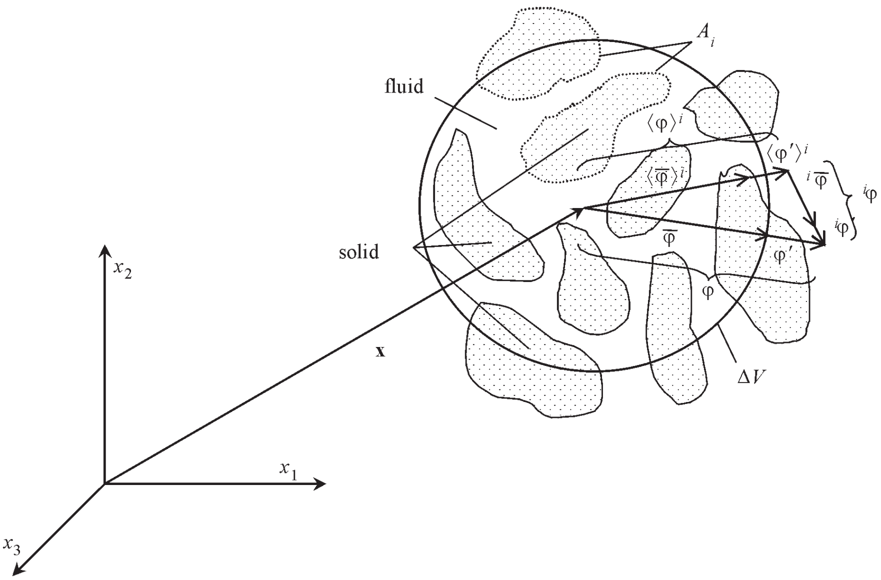

Fig. 1. REV, intrinsic average; space and time fluctuations.

details on the theorem of local volumetric average can be found in [5,17,18]. For single-phase flow, phase f is the fluid itself and $\mathbf{u}_i = 0$ if the porous substrate is assumed to be fixed. In developing Eqs. (3)–(5), the only restriction applied is the independence of ΔV in relation to time and space. If the medium is further assumed to be rigid, then ΔV_{f} is dependent only on space and also not time-dependent [17].

2.2. Time-average

The need for considering time fluctuations occurs when turbulence effects are of concern. The microscopic time-averaged equations are obtained from the instantaneous microscopic equations. For that, the time-average value of property, φ, associated with the fluid is given as

$$\bar{\varphi} = \frac{1}{\Delta t} \int_t^{t+\Delta t} \varphi \, \mathrm{d}t, \tag{6}$$

where Δt is the integration time interval. The instantaneous property φ can be defined as the sum of the time average, $\bar{\varphi}$, plus the fluctuating component, φ'

$$\varphi = \bar{\varphi} + \varphi' \tag{7}$$

being $\overline{\varphi'} = 0$.

2.3. Commutative properties

From the definition of volume-average (1) and time-average (6), one can conclude that the time-average of the volume-average of property φ is given by

$$\overline{\langle\varphi\rangle^v} = \frac{1}{\Delta t} \int_t^{t+\Delta t} \left[\frac{1}{\Delta V} \int_{\Delta V_{\mathrm{f}}} \varphi \, \mathrm{d}V \right] \mathrm{d}t. \tag{8}$$

The volume-average of the time-average is

$$\langle\bar{\varphi}\rangle^v = \frac{1}{\Delta V} \int_{\Delta V_{\mathrm{f}}} \left[\frac{1}{\Delta t} \int_t^{t+\Delta t} \varphi \, \mathrm{d}t \right] \mathrm{d}V. \tag{9}$$

As mentioned, for a rigid medium, the volume of fluid, ΔV_{f}, will be dependent only on space and not on time. If the time interval chosen for temporal averaging, Δt, is the same for all REV, then the volumetric average commutes with time-average because both integration domains in (8) and (9) are independent of each other. In this case, the order of application of average operators is immaterial so that Eqs. (8) and (9) will lead to

$$\overline{\langle\varphi\rangle^v} = \langle\bar{\varphi}\rangle^v \text{ or } \overline{\langle\varphi\rangle^i} = \langle\bar{\varphi}\rangle^i. \tag{10}$$

2.4. Double decomposition – space and time fluctuations

Fig. 1 shows that for any point located at certain position $\mathbf{x}$, surrounded by a volume ΔV, a volume-average can be defined. This value will be different depending on the selected volume ΔV. Also, for this very same entity (point $\mathbf{x}$), a time-average can be set, according to (6), being sole dependent on the time interval Δt. Further, from definition (1) and (7) one arrives at

$$\langle\varphi\rangle^i = \frac{1}{\Delta V_{\mathrm{f}}} \int_{\Delta V_{\mathrm{f}}} \varphi \, \mathrm{d}V = \frac{1}{\Delta V_{\mathrm{f}}} \int_{\Delta V_{\mathrm{f}}} \left(\bar{\varphi} + \varphi'\right) \mathrm{d}V = \langle\bar{\varphi}\rangle^i + \langle\varphi'\rangle^i \tag{11}$$

and combining Eqs. (2) and (6) one gets,

$$\bar{\varphi} = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} \varphi \, \mathrm{d}t = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} (\langle\varphi\rangle^{i} + {}^{i}\varphi)\, \mathrm{d}t = \overline{\langle\varphi\rangle^{i}} + \overline{{}^{i}\varphi}. \tag{12}$$

Further, the space-averaged value $\langle\varphi\rangle^{i}$ can also be decomposed into a time mean and fluctuating component as

$$\langle\varphi\rangle^{i} = \overline{\langle\varphi\rangle^{i}} + \langle\varphi\rangle^{i'}. \tag{13}$$

Using now the fact that the averages commute (Eq. (10)), a comparison of Eqs. (11) and (13) validates the following relationship

$$\langle\varphi'\rangle^{i} = \langle\varphi\rangle^{i'} \tag{14}$$

Eq. (14) means that *the volume-average of the time varying component is equal to the time fluctuation of the volume-average.* Similarly, if we consider the time-average component having also a spatial distribution, one has,

$$\bar{\varphi} = \left\langle\bar{\varphi}\right\rangle^{i} + {}^{i}\bar{\varphi}. \tag{15}$$

Likewise, a comparison between (12) and (15), in light of (10), gives,

$${}^{i}\bar{\varphi} = \overline{{}^{i}\varphi} \tag{16}$$

or, say, *the spatial deviation of the time-average quantity is equal to the time average of the spatial deviation*.

Further, since both time and space decompositions are based on the same value for φ, one can promptly write,

$$\varphi = \langle\varphi\rangle^{i} + {}^{i}\varphi = \bar{\varphi} + \varphi'. \tag{17}$$

Applying now full double decomposition to all terms on (17), one gets for φ,

$$\varphi = \underbrace{\langle\bar{\varphi}\rangle^{i} + \langle\varphi'\rangle^{i}}_{\langle\varphi\rangle^{i}} + \overbrace{{}^{i}\bar{\varphi} + {}^{i}\varphi'}^{{}^{i}\varphi} = \underbrace{\overline{\langle\varphi\rangle^{i}} + \overline{{}^{i}\varphi}}_{\bar{\varphi}} + \overbrace{\langle\varphi\rangle^{i'} + {}^{i}\varphi'}^{\varphi'}. \tag{18}$$

Here, it is interesting to note the meaning of the last term on each side of (18). The first term, ${}^{i}\varphi'$, is *the time fluctuation of the spatial component* whereas φ' means *the spatial component of the time varying term.* If, however, one make use of relationships (10), (14) and (16) to simplify (18), one finally concludes,

$${}^{i}\varphi' = {}^{i}\varphi' \tag{19}$$

and, for simplicity of notation, one can write both superscripts at the same level in the format: ${}^{i}\varphi'$.

Taking now the time-average of the fluctuating component, written into the form

$$\varphi' = \langle\varphi\rangle^{i'} + {}^{i}\varphi' = \langle\varphi'\rangle^{i} + {}^{i}\varphi' \tag{20}$$

gives further $\overline{{}^{i}\varphi'} = 0$. Likewise, volume averaging the spatial component, written as

$${}^{i}\varphi = {}^{i}\bar{\varphi} + {}^{i}\varphi' = \overline{{}^{i}\varphi} + {}^{i}\varphi' \tag{21}$$

will result in $\langle{}^{i}\varphi'\rangle^{i} = 0$.

With these ideas in mind, integration of local (microscopic) flow governing equations applied to the domain in Fig. 1 can be more easily treated. In addition, one can show that the order of integration (space and time) of these equation is, in fact, immaterial.

3. Mean flow equations

The development to follow assumes single-phase flow in a saturated, rigid porous medium (ΔV_{f} independent of time) for which, in accordance with (10), time-average operation on variable φ commutes with space average. Application of the double decomposition idea in Eq. (18) to the term inertia term in the momentum equation lead to four different terms. Not all these terms are considered in the same analysis in the literature.

3.1. Continuity

The microscopic continuity equation for an incompressible fluid flowing in a clean (non-porous) domain is given by

$$\nabla\cdot\mathbf{u} = 0. \tag{22}$$

Expanding the velocity field in (22) using the double decomposition idea of (18) gives,

$$\nabla\cdot\mathbf{u} = \nabla\cdot(\langle\bar{\mathbf{u}}\rangle^{i} + \langle\mathbf{u}'\rangle^{i} + {}^{i}\bar{\mathbf{u}} + {}^{i}\mathbf{u}') = 0. \tag{23}$$

Applying both volume-average (4) and time-average (6)–(23) gives

$$\nabla\cdot\left(\phi\langle\bar{\mathbf{u}}\rangle^{i}\right) = 0. \tag{24}$$

For continuity equation, the averaging order is immaterial regarding the final result.

3.2. Momentum – one average operator

The microscopic momentum equation for a fluid with constant properties is given by the Navier–Stokes equation as

$$\rho\left[\frac{\partial\mathbf{u}}{\partial t} + \nabla\cdot(\mathbf{u}\mathbf{u})\right] = -\nabla p + \mu\nabla^{2}\mathbf{u} + \rho\mathbf{g}. \tag{25}$$

Its time-average using $\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$ gives

$$\rho\left[\frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla\cdot(\bar{\mathbf{u}}\bar{\mathbf{u}})\right] = -\nabla\bar{p} + \mu\nabla^2\bar{\mathbf{u}} + \nabla\cdot(-\rho\overline{\mathbf{u}'\mathbf{u}'}) + \rho\mathbf{g}, \tag{26}$$

where the stresses, $-\rho\overline{\mathbf{u}'\mathbf{u}'}$, are the well-known Reynolds stresses. On the other hand, the volumetric average of (25) using the theorem of local volumetric average (Eqs. (3)–(5)), results in

$$\rho\left[\frac{\partial}{\partial t}(\phi\langle\mathbf{u}\rangle^i) + \nabla\cdot[\phi\langle\mathbf{u}\mathbf{u}\rangle^i]\right] = -\nabla(\phi\langle p\rangle^i) + \mu\nabla^2(\phi\langle\mathbf{u}\rangle^i) + \phi\rho\mathbf{g} + \mathbf{R}, \tag{27}$$

where

$$\mathbf{R} = \frac{\mu}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\nabla\mathbf{u})\ \mathrm{d}S - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}p\ \mathrm{d}S \tag{28}$$

represents the total drag force per unit volume due to the presence of the porous matrix, being composed by both viscous drag and form (pressure) drags. Further, using (2) to write $\mathbf{u} = \langle\mathbf{u}\rangle^i + {}^i\mathbf{u}$ in the inertia term,

$$\rho\left[\frac{\partial}{\partial t}(\phi\langle\mathbf{u}\rangle^i) + \nabla\cdot[\phi\langle\mathbf{u}\rangle^i\langle\mathbf{u}\rangle^i]\right] = -\nabla(\phi\langle p\rangle^i) + \mu\nabla^2(\phi\langle\mathbf{u}\rangle^i) - \nabla\cdot[\phi\langle{}^i\mathbf{u}{}^i\mathbf{u}\rangle^i] + \phi\rho\mathbf{g} + \mathbf{R}. \tag{29}$$

Hsu and Cheng [19] points that the third term on the right-hand side of (29), $\nabla\cdot(\phi\langle{}^i\mathbf{u}{}^i\mathbf{u}\rangle^i)$, represents the hydrodynamic dispersion due to spatial deviations. Note that Eq. (29) models typical porous media flow for $Re_p < 150$–200. When extending the analysis to turbulent flow, time varying quantities have to be considered.

3.3. Momentum equation – two average operators

The set of equations (26) and (29) are used when treating turbulent flow in clear fluid or low Re_p porous media flow, respectively. In each one of those equations only one averaging operator was applied, either time or volume, respectively. In this work, an investigation on the use of both operators are now conducted with the objective of modeling turbulent flow in porous media.

The volume average of (26) gives for the time mean flow in a porous medium, becomes

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right) + \nabla\cdot\left(\phi\langle\bar{\mathbf{u}}\bar{\mathbf{u}}\rangle^i\right)\right] = -\nabla\left(\phi\langle\bar{p}\rangle^i\right) + \mu\nabla^2\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right) + \nabla\cdot\left(-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i\right) + \phi\rho\mathbf{g} + \bar{\mathbf{R}}, \tag{30}$$

where

$$\bar{\mathbf{R}} = \frac{\mu}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\nabla\bar{\mathbf{u}})\ \mathrm{d}S - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\bar{p}\ \mathrm{d}S \tag{31}$$

is the time-averaged total drag force per unit volume ("body force"), due to solid particles, composed by both viscous and form (pressure) drags.

Likewise, applying now the time-average operation to (27), one gets

$$\rho\left[\frac{\partial}{\partial t}\overline{\left(\phi\langle\bar{\mathbf{u}}+\mathbf{u}'\rangle^i\right)} + \nabla\cdot\overline{\left(\phi\langle\left(\bar{\mathbf{u}}+\mathbf{u}'\right)\left(\bar{\mathbf{u}}+\mathbf{u}'\right)\rangle^i\right)}\right] = -\nabla\overline{\left(\phi\langle\bar{p}+p'\rangle^i\right)} + \mu\nabla^2\overline{\left(\phi\langle\bar{\mathbf{u}}+\mathbf{u}'\rangle^i\right)} + \phi\rho\mathbf{g} + \bar{\mathbf{R}}. \tag{32}$$

Dropping terms containing only one fluctuating quantity results in

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right) + \nabla\cdot\left(\phi\langle\bar{\mathbf{u}}\bar{\mathbf{u}}\rangle^i\right)\right] = -\nabla\left(\phi\langle\bar{p}\rangle^i\right) + \mu\nabla^2\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right) + \nabla\cdot\left(-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i\right) + \phi\rho\mathbf{g} + \bar{\mathbf{R}}, \tag{33}$$

where

$$\bar{\mathbf{R}} = \frac{\mu}{\Delta V}\int_{A_i}\mathbf{n}\cdot[\nabla\overline{(\bar{\mathbf{u}}+\mathbf{u}')}]\ \mathrm{d}S - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\overline{(\bar{p}+p')}\ \mathrm{d}S = \frac{\mu}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\nabla\bar{\mathbf{u}})\ \mathrm{d}S - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\bar{p}\ \mathrm{d}S. \tag{34}$$

Comparing (30) and (33) one can see that also for the momentum equation the order of the application of both averaging operators is immaterial.

It is interesting to emphasize that both views in the literature use the same final form for the momentum equation. The term $\overline{\mathbf{R}}$ is modeled by the Darcy–Forchheimer (Dupuit) expression after either order of application of the average operators. Since both orders of integration lead to the same equation, namely expression (31) or (34), there would be no reason for modeling them in a different form. Had the outcome of both integration processes been distinct, the use of a different model for each case would have been consistent. In fact, it has been pointed out by Pedras and de Lemos [20], that the major difference between those two paths lies in the definition of a suitable turbulent kinetic energy for the flow. Accordingly, the source of controversies comes from the inertia term, as seen below.

3.4. Inertia term – space and time (double) decomposition

Applying the double decomposition idea seen before for velocity (Eq. (18)), to the inertia term of (25) will lead to different sets of terms. In the literature, not all of them are used in the same analysis.

Starting with time decomposition and applying both average operators (see Eq. (30)) gives

$$\nabla\cdot\overline{\left(\phi\langle\mathbf{u}\mathbf{u}\rangle^i\right)} = \nabla\cdot\left(\overline{\phi\langle(\bar{\mathbf{u}}+\mathbf{u}')(\bar{\mathbf{u}}+\mathbf{u}')\rangle^i}\right) = \nabla\cdot\left[\phi\left(\langle\bar{\mathbf{u}}\bar{\mathbf{u}}\rangle^i + \langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i\right)\right]. \tag{35}$$

Using Eq. (15) to write $\bar{\mathbf{u}} = \langle\bar{\mathbf{u}}\rangle^i + {}^i\bar{\mathbf{u}}$ and plugging it into (35) gives

$$\begin{aligned}\nabla\cdot\left[\phi\left(\langle\bar{\mathbf{u}}\bar{\mathbf{u}}\rangle^i + \langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i\right)\right] &= \nabla\cdot\{\phi[\langle(\langle\bar{\mathbf{u}}\rangle^i + {}^i\bar{\mathbf{u}})(\langle\bar{\mathbf{u}}\rangle^i + {}^i\bar{\mathbf{u}})\rangle^i + \langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i]\}\\ &= \nabla\cdot\{\phi[\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i + \langle{}^i\bar{\mathbf{u}}{}^i\bar{\mathbf{u}}\rangle^i + \langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i]\}.\end{aligned} \tag{36}$$

Now, applying Eq. (20) to write $\mathbf{u}' = \langle\mathbf{u}'\rangle^i + {}^i\mathbf{u}'$ and substituting it into (36) gives

$$\begin{aligned}&\nabla\cdot\left\{\phi[\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i + \langle{}^i\bar{\mathbf{u}}{}^i\bar{\mathbf{u}}\rangle^i + \langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i]\right\}\\ &= \nabla\cdot\left\{\phi[\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i + \langle{}^i\bar{\mathbf{u}}{}^i\bar{\mathbf{u}}\rangle^i + \overline{\langle(\langle\mathbf{u}'\rangle^i + {}^i\mathbf{u}')(\langle\mathbf{u}'\rangle^i + {}^i\mathbf{u}')\rangle^i}]\right\}\\ &= \nabla\cdot\left\{\phi[\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i + \langle{}^i\bar{\mathbf{u}}{}^i\bar{\mathbf{u}}\rangle^i\right.\\ &\qquad\left. + \overline{\langle(\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i + \langle\mathbf{u}'\rangle^i{}^i\mathbf{u}' + {}^i\mathbf{u}'\langle\mathbf{u}'\rangle^i + {}^i\mathbf{u}'{}^i\mathbf{u}')\rangle^i}]\right\}\\ &= \nabla\cdot\left\{\phi[\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i + \langle{}^i\bar{\mathbf{u}}{}^i\bar{\mathbf{u}}\rangle^i + \overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}\right.\\ &\qquad\left. + \overline{\langle\langle\mathbf{u}'\rangle^i{}^i\mathbf{u}'\rangle^i} + \overline{\langle{}^i\mathbf{u}'\langle\mathbf{u}'\rangle^i\rangle^i} + \langle\overline{{}^i\mathbf{u}'{}^i\mathbf{u}'}\rangle^i]\right\}.\end{aligned} \tag{37}$$

The fourth and fifth terms on the right-hand side of (37) contains only one space varying quantity and will vanish under the application of volume integration. Eq. (37) will then be reduced to

$$\nabla\cdot\overline{\left(\phi\langle\mathbf{u}\mathbf{u}\rangle^i\right)} = \nabla\cdot\left\{\phi[\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i + \overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i} + \langle{}^i\bar{\mathbf{u}}{}^i\bar{\mathbf{u}}\rangle^i + \langle\overline{{}^i\mathbf{u}'{}^i\mathbf{u}'}\rangle^i]\right\}. \tag{38}$$

Using the equivalence (10), (14) and (16), Eq. (38) can be further rewritten as

$$\nabla\cdot\overline{\left(\phi\langle\mathbf{u}\mathbf{u}\rangle^i\right)} = \nabla\cdot\left\{\phi[\overline{\langle\mathbf{u}\rangle^i}\,\overline{\langle\mathbf{u}\rangle^i} + \overline{\langle\mathbf{u}\rangle^{i'}\langle\mathbf{u}\rangle^{i'}} + \langle\overline{{}^i\mathbf{u}}\,\overline{{}^i\mathbf{u}}\rangle^i + \langle\overline{{}^i\mathbf{u}'{}^i\mathbf{u}'}\rangle^i]\right\} \tag{39}$$

with an interpretation of the terms in (38) given later.

Another route to follow to reach the same results is to start out with the application of the space decomposition in the inertia term, as usually done in classical mathematical treatment of porous media flow analysis. Then one has [19]

$$\nabla\cdot\overline{\left(\phi\langle\mathbf{u}\mathbf{u}\rangle^i\right)} = \nabla\cdot\overline{\left(\phi\langle(\langle\mathbf{u}\rangle^i + {}^i\mathbf{u})(\langle\mathbf{u}\rangle^i + {}^i\mathbf{u})\rangle^i\right)} = \nabla\cdot\overline{[\phi\left(\langle\mathbf{u}\rangle^i\langle\mathbf{u}\rangle^i + \langle{}^i\mathbf{u}{}^i\mathbf{u}\rangle^i\right)]}. \tag{40}$$

The time-average of the right-hand side of (40), using Eq. (11) to express $\langle\mathbf{u}\rangle^i = \langle\bar{\mathbf{u}}\rangle^i + \langle\mathbf{u}'\rangle^i$, becomes

$$\begin{aligned}&\nabla\cdot\overline{[\phi\left(\langle\mathbf{u}\rangle^i\langle\mathbf{u}\rangle^i + \langle{}^i\mathbf{u}{}^i\mathbf{u}\rangle^i\right)]}\\ &= \nabla\cdot\left\{\overline{\phi[\left(\langle\bar{\mathbf{u}}\rangle^i + \langle\mathbf{u}'\rangle^i\right)\left(\langle\bar{\mathbf{u}}\rangle^i + \langle\mathbf{u}'\rangle^i\right) + \langle{}^i\mathbf{u}{}^i\mathbf{u}\rangle^i]}\right\}\\ &= \nabla\cdot\left\{\phi[\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i + \overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i} + \overline{\langle{}^i\mathbf{u}{}^i\mathbf{u}\rangle^i}]\right\}.\end{aligned} \tag{41}$$

With the help of Eq. (21) one can write ${}^i\mathbf{u} = {}^i\bar{\mathbf{u}} + {}^i\mathbf{u}'$ which, inserted into (41), gives,

$$\begin{aligned}&\nabla\cdot\left\{\phi[\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i + \overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i} + \overline{\langle{}^i\mathbf{u}{}^i\mathbf{u}\rangle^i}]\right\}\\ &= \nabla\cdot\left\{\phi[\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i + \overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i} + \overline{\langle({}^i\bar{\mathbf{u}} + {}^i\mathbf{u}')({}^i\bar{\mathbf{u}} + {}^i\mathbf{u}')\rangle^i}]\right\}\\ &= \nabla\cdot\left\{\phi[\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i + \overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}\right.\\ &\qquad\left. + \langle\overline{{}^i\bar{\mathbf{u}}{}^i\bar{\mathbf{u}}} + \overline{{}^i\bar{\mathbf{u}}{}^i\mathbf{u}'} + \overline{{}^i\mathbf{u}'{}^i\bar{\mathbf{u}}} + \overline{{}^i\mathbf{u}'{}^i\mathbf{u}'}\rangle^i]\right\}.\end{aligned} \tag{42}$$

Application of the time-average operator to the fourth and fifth terms on the right-hand side of (42), containing only one fluctuating component, vanishes it. In addition, remembering that with (14) there is the equivalence $\langle\mathbf{u}'\rangle^i = \langle\mathbf{u}\rangle^{i'}$, with (10) one can write $\overline{\langle\mathbf{u}\rangle^i} = \langle\bar{\mathbf{u}}\rangle^i$ and using (16) one has ${}^i\bar{\mathbf{u}} = \overline{{}^i\mathbf{u}}$, Eq. (42) becomes

$$\begin{aligned}&\nabla\cdot\overline{[\phi\left(\langle\mathbf{u}\rangle^i\langle\mathbf{u}\rangle^i + \langle{}^i\mathbf{u}{}^i\mathbf{u}\rangle^i\right)]}\\ &= \nabla\cdot\left\{\phi[\underbrace{\langle\bar{\mathbf{u}}\rangle^i\langle\bar{\mathbf{u}}\rangle^i}_{\mathrm{I}} + \underbrace{\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}}_{\mathrm{II}} + \underbrace{\langle{}^i\bar{\mathbf{u}}{}^i\bar{\mathbf{u}}\rangle^i}_{\mathrm{III}} + \underbrace{\langle\overline{{}^i\mathbf{u}'{}^i\mathbf{u}'}\rangle^i}_{\mathrm{IV}}]\right\},\end{aligned} \tag{43}$$

which is the same result of (38).

A physical significance of all four terms on the right-hand side of (43) can be discussed as: (I) Convective term of macroscopic mean velocity. (II) Turbulent (Reynolds) stresses divided by density ρ due to the fluctuating component of the macroscopic velocity. (III) *Dispersion* associated with spatial fluctuations of microscopic time mean velocity. Note that this term is also present in laminar flow, or say, when $Re_p < 150$. (IV) *Turbulent dispersion* in a porous medium due to both time and spatial fluctuations of the microscopic velocity.

4. Macroscopic Reynolds stress tensor

For clear fluid, the use of the eddy-diffusivity concept for expressing the *stress-rate of strain* relationship for the Reynolds stress appearing in (26) gives,

$$-\rho\overline{\mathbf{u}'\mathbf{u}'} = \mu_t 2\bar{\mathbf{D}} - \frac{2}{3}\rho k\mathbf{I}, \tag{44}$$

where $\bar{\mathbf{D}} = [\nabla\bar{\mathbf{u}} + (\nabla\bar{\mathbf{u}})^{\mathrm{T}}]/2$ is the mean deformation tensor, $k = \overline{\mathbf{u}'\cdot\mathbf{u}'}/2$ is the turbulent kinetic energy per unit mass and **I** is the unity tensor. Applying (44) to (26) gives further

$$\rho\left[\frac{\partial\bar{\mathbf{u}}}{\partial t}+\nabla\cdot\left(\bar{\mathbf{u}}\bar{\mathbf{u}}\right)\right]=-\nabla\left(\bar{p}+\frac{2}{3}\rho k\right)+\mu\nabla^2\bar{\mathbf{u}}+\nabla\cdot\left(\mu_t 2\bar{\mathbf{D}}\right)+\rho\mathbf{g}. \quad (45)$$

In order to obtain an equivalent expression for the macroscopic Reynolds stress tensor, the volume-averaging operator with respect to ΔV will be carried out in both Eqs. (26) and (45). Making use of the theorem of local volumetric average (Eqs. (3)–(5)), the several terms in Eqs. (26) and (45) become

$$\left\langle\frac{\partial\bar{\mathbf{u}}}{\partial t}\right\rangle^v=\frac{\partial}{\partial t}\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right), \quad (46)$$

$$\left\langle\nabla\cdot\left(\bar{\mathbf{u}}\bar{\mathbf{u}}\right)\right\rangle^v=\nabla\cdot\left(\phi\langle\bar{\mathbf{u}}\bar{\mathbf{u}}\rangle^i\right)+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\left(\bar{\mathbf{u}}\bar{\mathbf{u}}\right)\,\mathrm{d}S, \quad (47)$$

$$\langle\nabla\bar{p}\rangle^v=\nabla\left(\phi\langle\bar{p}\rangle^i\right)+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\bar{p}\,\mathrm{d}S, \quad (48)$$

$$\langle\nabla\cdot\nabla\bar{\mathbf{u}}\rangle^v=\nabla\cdot\langle\nabla\bar{\mathbf{u}}\rangle^v+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\nabla\bar{\mathbf{u}})\,\mathrm{d}S=\nabla^2\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)+\nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\bar{\mathbf{u}}\,\mathrm{d}S\right]+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\nabla\bar{\mathbf{u}})\,\mathrm{d}S, \quad (49)$$

$$\langle\rho\mathbf{g}\rangle^v=\phi\rho\mathbf{g}. \quad (50)$$

Eq. (26) has further

$$\langle\nabla\cdot\overline{\mathbf{u}'\mathbf{u}'}\rangle^v=\nabla\cdot\left(\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i\right)+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\overline{\mathbf{u}'\mathbf{u}'}\,\mathrm{d}S \quad (51)$$

and Eq. (45),

$$\left\langle\nabla\left(\frac{2}{3}\rho k\right)\right\rangle^v=\nabla\left(\frac{2}{3}\rho\phi\langle k\rangle^i\right)+\frac{2}{3}\frac{\rho}{\Delta V}\int_{A_i}\mathbf{n}k\,\mathrm{d}S, \quad (52)$$

$$\left\langle\nabla\cdot\left(\mu_t 2\bar{\mathbf{D}}\right)\right\rangle^v=\nabla\cdot\left(\mu_{t_\phi}2\langle\bar{\mathbf{D}}\rangle^v\right)+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\left(\mu_t 2\bar{\mathbf{D}}\right)\,\mathrm{d}S, \quad (53)$$

where

$$\mu_{t_\phi}\langle\bar{\mathbf{D}}\rangle^v=\langle\mu_t\bar{\mathbf{D}}\rangle^v \quad (54)$$

and

$$\langle\bar{\mathbf{D}}\rangle^v=\frac{1}{2}\langle\nabla\bar{\mathbf{u}}+(\nabla\bar{\mathbf{u}})^{\mathrm{T}}\rangle^v=\frac{1}{2}\left\{\left[\nabla\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)+[\nabla\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)]^{\mathrm{T}}\right]+\frac{1}{\Delta V}\int_{A_i}[\mathbf{n}\bar{\mathbf{u}}+(\mathbf{n}\bar{\mathbf{u}})^{\mathrm{T}}]\,\mathrm{d}S\right\}. \quad (55)$$

Noting that at the interface, $A_i, \bar{\mathbf{u}}=\overline{\mathbf{u}'\mathbf{u}'}=\mu_t=k=0$ an equation for the macroscopic momentum equation for turbulent flow in porous media based on (26) is,

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)+\nabla\cdot\left(\phi\langle\bar{\mathbf{u}}\bar{\mathbf{u}}\rangle^i\right)\right]=-\nabla\left(\phi\langle\bar{p}\rangle^i\right)+\mu\nabla^2\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)+\nabla\cdot\left(-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i\right)+\phi\rho\mathbf{g}+\bar{\mathbf{R}} \quad (56)$$

and based on (45),

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)+\nabla\cdot\left(\phi\langle\bar{\mathbf{u}}\bar{\mathbf{u}}\rangle^i\right)\right]=-\nabla\left(\phi\langle\bar{p}\rangle^i+\frac{2}{3}\phi\rho\langle k\rangle^i\right)+\mu\nabla^2\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)+\nabla\cdot(\mu_{t_\phi}2\langle\bar{\mathbf{D}}\rangle^v)+\phi\rho\mathbf{g}+\bar{\mathbf{R}}, \quad (57)$$

where $\bar{\mathbf{R}}$ is given by (31). Further, the term $-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i$ in (56) is the macroscopic Reynolds stress tensor and the deformation tensor in (57) reads,

$$\langle\bar{\mathbf{D}}\rangle^v=\frac{1}{2}\left[\nabla\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)+\left[\nabla\left(\phi\langle\bar{\mathbf{u}}\rangle^i\right)\right]^{\mathrm{T}}\right]. \quad (58)$$

Comparing now Eqs. (56) and (57), a proposal for the macroscopic Reynolds stress tensor can be made as

$$-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i=\mu_{t_\phi}2\langle\bar{\mathbf{D}}\rangle^v-\frac{2}{3}\phi\rho\langle k\rangle^i\mathbf{I} \quad (59)$$

which is similar to the eddy-diffusivity for microscopic flow embodied in Eq. (44). Note, however, that the coefficient μ_{t_ϕ} appearing in (59) is defined according to (54) and is not necessarily the same coefficient appearing for clear fluid flow used in (44). In this work, for simplicity, an expression of the type $\mu_{t_\phi}=\rho c_\mu\langle k\rangle^{i^2}/\langle\varepsilon\rangle^i$ was made use of.

The macroscopic Reynolds stresses tensor of Eq. (30), modeled herein by (59), can be further expanded with the help of $\mathbf{u}'=\langle\mathbf{u}'\rangle^i+{}^i\mathbf{u}'$ as

$$-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i=-\rho\phi\left[\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}+\langle\overline{{}^i\mathbf{u}'{}^i\mathbf{u}'}\rangle^i\right]. \quad (60)$$

The first term on the right-hand side is associated with time fluctuations of the macroscopic mean velocity whereas the second one represents the turbulent dispersion in porous medium due to both time and spatial fluctuations of the microscopic velocity (see term (IV) in (43)). Recent turbulence models presented in literature [8–10] do not consider the second term on the right-hand side of (60).

Also interesting to note is the intrinsic (fluid) average for k given here as $\langle k\rangle^i$ and appearing for the first time in (52). The kinetic energy used in [8–10] differs from $\langle k\rangle^i$ and is given by $k_m=\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}/2$. Recently, Pedras and de Lemos [21] have shown the relationship between these two quantities as being

$$\langle k\rangle^i = \langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2 = \overline{\langle\mathbf{u}'\rangle^i\cdot\langle\mathbf{u}'\rangle^i}/2 + \langle\overline{{}^i\mathbf{u}'\cdot{}^i\mathbf{u}'}\rangle^i/2 = k_m + \langle\overline{{}^i\mathbf{u}'\cdot{}^i\mathbf{u}'}\rangle^i/2. \quad (61)$$

The last term on the right-hand side of (61) is the extra turbulent kinetic energy obtained by adding up elements of the main diagonal of term (IV) in Eq. (43). As seen, models based on k_m do not account for all of the turbulent kinetic energy associated with the flow.

5. The $k\varepsilon$ equations for porous media

5.1. Model equation for $\langle k\rangle^i$

An equation for the intrinsic average for the TKE, $\langle k\rangle^i$, is obtained by applying the volume-average operator (1) to the transport equation for k. An equation for k, in turn, can be readily obtained in a number of works in the literature (e.g., [22]) as

$$\rho\left[\frac{\partial k}{\partial t} + \nabla\cdot(\bar{\mathbf{u}}k)\right] = -\rho\nabla\cdot\overline{\left[\mathbf{u}'\left(\frac{p'}{\rho}+k\right)\right]} + \mu\nabla^2 k - \rho\overline{\mathbf{u}'\mathbf{u}'}:\nabla\bar{\mathbf{u}} - \rho\varepsilon, \quad (62)$$

where $\varepsilon = \mu\overline{\nabla\mathbf{u}':(\nabla\mathbf{u}')^{\mathrm{T}}}/\rho$ is the dissipation rate of k (this interpretation given to ε being strictly correct only for isotropic turbulence). Noting that the Laplacian of k in Eq. (62) can be rewritten as

$$\nabla\cdot\nabla k = \frac{1}{2}\nabla\cdot\nabla(\overline{\mathbf{u}'\cdot\mathbf{u}'}) = \nabla\cdot\overline{\left[\mathbf{u}'\cdot(\nabla\mathbf{u}')^{\mathrm{T}}\right]}. \quad (63)$$

and taking the volumetric average of (62) with respect to ΔV, one has for individual terms:

$$\left\langle\frac{\partial k}{\partial t}\right\rangle^v = \frac{\partial}{\partial t}\left(\phi\langle k\rangle^i\right), \quad (64)$$

$$\langle\nabla\cdot(\bar{\mathbf{u}}k)\rangle^v = \nabla\cdot\left(\phi\langle\bar{\mathbf{u}}k\rangle^i\right) + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\bar{\mathbf{u}}k)\ \mathrm{d}S, \quad (65)$$

$$\left\langle\nabla\cdot\overline{\left[\mathbf{u}'\left(\frac{p'}{\rho}+k\right)\right]}\right\rangle^v = \nabla\cdot\left\{\phi\left\langle\overline{\mathbf{u}'\left(\frac{p'}{\rho}+k\right)}\right\rangle^i\right\} + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\overline{\left[\mathbf{u}'\left(\frac{p'}{\rho}+k\right)\right]}\ \mathrm{d}S, \quad (66)$$

$$\begin{aligned}\langle\nabla^2 k\rangle^v &= \left\langle\nabla\cdot\overline{[\mathbf{u}'\cdot(\nabla\mathbf{u}')^{\mathrm{T}}]}\right\rangle^v = \nabla\cdot\left\langle\overline{[\mathbf{u}'\cdot(\nabla\mathbf{u}')^{\mathrm{T}}]}\right\rangle^v + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\overline{[\mathbf{u}'\cdot(\nabla\mathbf{u}')^{\mathrm{T}}]}\,\mathrm{d}S \\ &= \nabla\cdot\langle\nabla k\rangle^v + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\overline{[\mathbf{u}'\cdot(\nabla\mathbf{u}')^{\mathrm{T}}]}\,\mathrm{d}S \\ &= \nabla^2\left(\phi\langle k\rangle^i\right) + \nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}k\ \mathrm{d}S\right] + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\overline{[\mathbf{u}'\cdot(\nabla\mathbf{u}')^{\mathrm{T}}]}\ \mathrm{d}S,\end{aligned} \quad (67)$$

$$\rho\langle\overline{\mathbf{u}'\mathbf{u}'}:\nabla\bar{\mathbf{u}}\rangle^v = \rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}:\nabla\bar{\mathbf{u}}\rangle^i, \quad (68)$$

$$\rho\langle\varepsilon\rangle^v = \rho\phi\langle\varepsilon\rangle^i. \quad (69)$$

Noting further that at interface $A_i, \bar{\mathbf{u}} = \mathbf{u}' = k = 0$, a transport $\langle k\rangle^i$ equation for becomes

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle k\rangle^i\right) + \nabla\cdot\left(\phi\langle\bar{\mathbf{u}}k\rangle^i\right)\right] = -\rho\nabla\cdot\left\{\phi\left\langle\overline{\mathbf{u}'\left(\frac{p'}{\rho}+k\right)}\right\rangle^i\right\} + \mu\nabla^2\left(\phi\langle k\rangle^i\right) - \rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}:\nabla\bar{\mathbf{u}}\rangle^i - \rho\phi\langle\varepsilon\rangle^i. \quad (70)$$

5.1.1. Turbulent diffusion

The first term on the right-hand side of (70) represents the turbulent diffusion of $\phi\langle k\rangle^i$ due to pressure fluctuations. This term is usually modeled in the literature by a gradient diffusion like expression as

$$-\rho\nabla\cdot\left\{\phi\left\langle\overline{\mathbf{u}'\left(\frac{p'}{\rho}+k\right)}\right\rangle^i\right\} = \rho\nabla\cdot\left[\frac{\mu_{t_\phi}}{\rho\sigma_k}\nabla\left(\phi\langle k\rangle^i\right)\right]. \quad (71)$$

5.1.2. Dispersion and production

The second term on the right-hand side of (70) can be expanded as

$$\nabla\cdot\left(\phi\langle\bar{\mathbf{u}}k\rangle^i\right) = \nabla\cdot\left[\phi\left(\langle\bar{\mathbf{u}}\rangle^i\langle k\rangle^i + \langle{}^i\bar{\mathbf{u}}{}^i k\rangle^i\right)\right]. \quad (72)$$

The first term on the right-hand side of (72) is the convection of $\langle k\rangle^i$ due to the macroscopic velocity whereas the second one is the convective transport due to spatial deviations of both k and $\mathbf{u}$. Likewise, the production term on the right-hand side of (70) can be expanded as

$$-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}:\nabla\bar{\mathbf{u}}\rangle^i = -\rho\phi[\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i:\langle\nabla\bar{\mathbf{u}}\rangle^i + \langle{}^i(\overline{\mathbf{u}'\mathbf{u}'}):{}^i(\nabla\bar{\mathbf{u}})\rangle^i]. \quad (73)$$

Similarly, the first term on the right-hand side of (73) is the production of $\langle k\rangle^i$ due to the mean macroscopic flow and the second one is the $\langle k\rangle^i$ production associated with spatial deviations of flow quantities k and $\mathbf{u}$.

The extra terms appearing in Eqs. (72) and (73), respectively, represent extra transport/production of $\langle k\rangle^i$ due to the presence of solid material inside the integration volume. They should be null for the limiting case of clear fluid flow, or say, when $\phi\to 1 \Rightarrow K\to\infty$. Also, they should be proportional to the macroscopic velocity and to $\langle k\rangle^i$ itself.

In this work, a proposal for those two extra transport/production rates of $\langle k\rangle^i$ is given as

$$\nabla\cdot\left(\phi\langle{}^i\bar{\mathbf{u}}^i k\rangle^i\right)-\rho\phi\langle{}^i(\overline{\mathbf{u}'\mathbf{u}'}):{}^i(\nabla\bar{\mathbf{u}})\rangle^i=c_k\rho\phi\frac{\langle k\rangle^i|\bar{\mathbf{u}}_\mathrm{D}|}{\sqrt{K}}, \tag{74}$$

where c_k is a non-dimensional constant. Using further the Dupuit–Forchheimer relationship $\bar{\mathbf{u}}_\mathrm{D}=\phi\langle\bar{\mathbf{u}}\rangle^\mathbf{i}$ and the model in Eq. (74), the transport equation for $\langle k\rangle^i$ becomes

$$\begin{aligned}\rho\left[\frac{\partial}{\partial t}\left(\phi\langle k\rangle^i\right)+\nabla\cdot\left(\bar{\mathbf{u}}_\mathrm{D}\langle k\rangle^i\right)\right]&\\=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_k}\right)\nabla\left(\phi\langle k\rangle^i\right)\right]-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i:\nabla\bar{\mathbf{u}}_\mathrm{D}&\\+c_k\rho\phi\frac{\langle k\rangle^i|\bar{\mathbf{u}}_\mathrm{D}|}{\sqrt{K}}-\rho\phi\langle\varepsilon\rangle^i,&\end{aligned} \tag{75}$$

where $\rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i$ is given by (59) and σ_k is an empirical constant.

As seen, in the present work the influence of the porous matrix on the level of $\langle k\rangle^i$ is considered by Eq. (74). As also mentioned, in the limiting case of clear flow ($\phi\to 1\Rightarrow K\to\infty$) this extra generation rate added to (75) should vanish. Under this condition, $\langle k\rangle^i$ should resemble k and the transport equation for the turbulent kinetic energy in clear fluid should be recovered. Also, the extra term included in Eq. (75) determines the rate of production of $\langle k\rangle^i$ due to the presence of a porous matrix. For a fixed value of the Darcy velocity $\bar{\mathbf{u}}_\mathrm{D}$ through a porous bed, the amount of mechanical energy converted into turbulence should depend on the medium properties. For the limiting case of high porosity and permeability media ($\phi\to 1\Rightarrow K\to\infty$), and for one-dimensional, fully developed flow with a flat velocity profile, no fraction of this available mechanical energy is expected to generate turbulence. The flow, in this situation, behaves like clear fluid flow. Consequently, no turbulence is generated and $\langle k\rangle^i$, if existing at inlet, decays to zero along the flow. As the flow resistance increases, by increasing $\phi/\sqrt{K}$, gradients of local $\mathbf{u}$ within the pore will contribute to increasing $\langle k\rangle^i$. The proposed form for this extra production term, given here by (74), is consistent with this expected behavior for $\langle k\rangle^i$.

5.2. Model equation for $\langle\varepsilon\rangle^i$

In a similar manner, a transport equation for $\langle k\rangle^i$ is obtained by applying the volume-average operator (1), to a transport equation of the form [23]

$$\begin{aligned}\rho\left[\frac{\partial\varepsilon}{\partial t}+\frac{\partial(\bar{u}_k\varepsilon)}{\partial x_k}\right]&=-2\mu\frac{\partial\bar{u}_j}{\partial x_k}\left\{\overline{\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_k}{\partial x_l}}+\overline{\frac{\partial u'_i}{\partial x_j}\frac{\partial u'_i}{\partial x_k}}\right\}\\&-2\mu\left\{\overline{\frac{\partial u'_j}{\partial x_k}\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_k}{\partial x_l}}+\frac{\mu}{\rho}\overline{\left(\frac{\partial^2u'_j}{\partial x_k\partial x_l}\right)^2}\right\}\\&-\mu\frac{\partial}{\partial x_k}\left\{\overline{u'_k\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_j}{\partial x_l}}+\frac{2}{\rho}\overline{\frac{\partial u'_k}{\partial x_l}\frac{\partial p'}{\partial x_l}}\right\}\\&-2\mu\frac{\partial^2\bar{u}_j}{\partial x_l\partial x_k}\left(\overline{u'_k\frac{\partial u'_j}{\partial x_l}}\right)+\mu\frac{\partial^2\varepsilon}{\partial x_l\partial x_l}.\end{aligned} \tag{76}$$

The volumetric average of (76) with respect to ΔV gives for each term:

$$\left\langle\frac{\partial\varepsilon}{\partial t}\right\rangle^v=\frac{\partial}{\partial t}\left(\phi\langle\varepsilon\rangle^i\right), \tag{77}$$

$$\left\langle\frac{\partial(\bar{u}_k\varepsilon)}{\partial x_k}\right\rangle^v=\frac{\partial\left(\phi\langle\bar{u}_k\varepsilon\rangle^i\right)}{\partial x_k}+\frac{1}{\Delta V}\int_{A_i}n_k(\bar{u}_k\varepsilon)\ \mathrm{d}S, \tag{78}$$

$$\begin{aligned}&\left\langle 2\mu\frac{\partial\bar{u}_j}{\partial x_k}\left\{\overline{\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_k}{\partial x_l}}+\overline{\frac{\partial u'_i}{\partial x_j}\frac{\partial u'_i}{\partial x_k}}\right\}\right\rangle^v\\&=\phi\left\langle 2\mu\frac{\partial\bar{u}_j}{\partial x_k}\left\{\overline{\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_k}{\partial x_l}}+\overline{\frac{\partial u'_i}{\partial x_j}\frac{\partial u'_i}{\partial x_k}}\right\}\right\rangle^i,\end{aligned} \tag{79}$$

$$\begin{aligned}&\left\langle 2\mu\left\{\overline{\frac{\partial u'_j}{\partial x_k}\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_k}{\partial x_l}}+\frac{\mu}{\rho}\overline{\left(\frac{\partial^2u'_j}{\partial x_k\partial x_l}\right)^2}\right\}\right\rangle^v\\&=\phi\left\langle 2\mu\left\{\overline{\frac{\partial u'_j}{\partial x_k}\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_k}{\partial x_l}}+\frac{\mu}{\rho}\overline{\left(\frac{\partial^2u'_j}{\partial x_k\partial x_l}\right)^2}\right\}\right\rangle^i,\end{aligned} \tag{80}$$

$$\begin{aligned}&\left\langle\mu\frac{\partial}{\partial x_k}\left\{\overline{u'_k\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_j}{\partial x_l}}+\frac{2}{\rho}\overline{\frac{\partial u'_k}{\partial x_l}\frac{\partial p'}{\partial x_l}}\right\}\right\rangle^v\\&=\phi\left\langle\mu\frac{\partial}{\partial x_k}\left\{\overline{u'_k\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_j}{\partial x_l}}+\frac{2}{\rho}\overline{\frac{\partial u'_k}{\partial x_l}\frac{\partial p'}{\partial x_l}}\right\}\right\rangle^i,\end{aligned} \tag{81}$$

$$\left\langle 2\mu\frac{\partial^2\bar{u}_j}{\partial x_l\partial x_k}\left(\overline{u'_k\frac{\partial u'_j}{\partial x_l}}\right)\right\rangle^v=\phi\left\langle 2\mu\frac{\partial^2\bar{u}_j}{\partial x_l\partial x_k}\left(\overline{u'_k\frac{\partial u'_j}{\partial x_l}}\right)\right\rangle^i, \tag{82}$$

$$\left\langle\mu\frac{\partial^2\varepsilon}{\partial x_l\partial x_l}\right\rangle^v=\phi\left\langle\mu\frac{\partial^2\varepsilon}{\partial x_l\partial x_l}\right\rangle^i. \tag{83}$$

Again, noting that at the interface, $A_i,\bar{\mathbf{u}}=0$, an equation for can be expressed as

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle\varepsilon\rangle^i\right)+\frac{\partial\left(\phi\langle\bar{u}_k\rangle^i\langle\varepsilon\rangle^i\right)}{\partial x_k}\right]$$
$$=-\phi\left\langle 2\mu\frac{\partial\bar{u}_j}{\partial x_k}\left\{\overline{\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_k}{\partial x_l}}+\overline{\frac{\partial u'_l}{\partial x_j}\frac{\partial u'_l}{\partial x_k}}\right\}\right\rangle^i$$
$$-\phi\left\langle 2\mu\left\{\overline{\frac{\partial u'_j}{\partial x_k}\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_k}{\partial x_l}}+\frac{\mu}{\rho}\overline{\left(\frac{\partial^2 u'_j}{\partial x_k\partial x_l}\right)^2}\right\}\right\rangle^i$$
$$-\phi\left\langle\mu\frac{\partial}{\partial x_k}\left\{\overline{u'_k\frac{\partial u'_j}{\partial x_l}\frac{\partial u'_j}{\partial x_l}}+\frac{2}{\rho}\overline{\frac{\partial u'_k}{\partial x_l}\frac{\partial p'}{\partial x_l}}\right\}\right\rangle^i$$
$$-\phi\left\langle 2\mu\frac{\partial^2\bar{u}_j}{\partial x_l\partial x_k}\left(\overline{u'_k\frac{\partial u'_j}{\partial x_l}}\right)\right\rangle^i+\phi\left\langle\mu\frac{\partial^2\varepsilon}{\partial x_l\partial x_l}\right\rangle^i$$
$$-\rho\frac{\partial\left(\phi\langle{}^i\bar{u}_k{}^i\varepsilon\rangle^i\right)}{\partial x_k}. \qquad (84)$$

Eq. (84) is composed by terms considering local rate of change, convection, dispersion, diffusion (molecular plus turbulent) and generation/destruction rates of $\langle\varepsilon\rangle^i$. Making use of the Dupuit–Forchheimer relationship, $\bar{\mathbf{u}}_D=\phi\langle\bar{\mathbf{u}}\rangle^{\mathbf{i}}$, a model for it can be proposed as

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle\varepsilon\rangle^i\right)+\nabla\cdot\left(\bar{\mathbf{u}}_D\langle\varepsilon\rangle^i\right)\right]$$
$$=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_\varepsilon}\right)\nabla\left(\phi\langle\varepsilon\rangle^i\right)\right]+c_{1\varepsilon}(-\rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i$$
$$:\nabla\bar{\mathbf{u}}_D)\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i}+c_{2\varepsilon}\rho\phi\left\{c_k\frac{\langle\varepsilon\rangle^i|\bar{\mathbf{u}}_D|}{\sqrt{K}}-\frac{\langle\varepsilon\rangle^{i^2}}{\langle k\rangle^i}\right\}, \qquad (85)$$

where $\sigma_\varepsilon, c_{1\varepsilon}$ and $c_{2\varepsilon}$ are constants. As with the case of $\langle k\rangle^i$, the overall dissipation rate of $\langle\varepsilon\rangle^i$, the last term on the right-hand side of (85), contains an additional factor that is dependent on the porous substrate. This additional term vanishes for the limiting case of clear fluid ($\phi\to 1 => K\to\infty$). In addition, for macroscopic fully developed uni-dimensional flow in isotropic and homogeneous media, the production rate of $\langle\varepsilon\rangle^i$ will be solely due to spatial deviations within the REV and will be totally dissipated within the same domain. These ideas are used below when determining a numerical value for the introduced constant c_k.

6. Results and discussion

6.1. Microscopic computations

The equations for the microscopic flow were numerically solved inside the elementary cell of Fig. 2 using the computational grid presented in Fig. 2. This geometry was used in order to represent the porous matrix as an infinite periodic array. The Reynolds number Re_H based on the cell height H was varied from nearly 0.35 (creeping flow) to 1.2×10^6 (fully turbulent regime). A version of the k–ε model for low Re flow was also incorporated in the code developed, following the damping functions presented by Abe et al. [24]. The non-orthogonal grid was based on a generalized coordinate system, leading to irregular control-volumes in a total of 150×100 for the high Re model and 300×200 for the low Re cases. For $Re_H=1.2\times10^5$, both k–ε models were calculated for comparison.

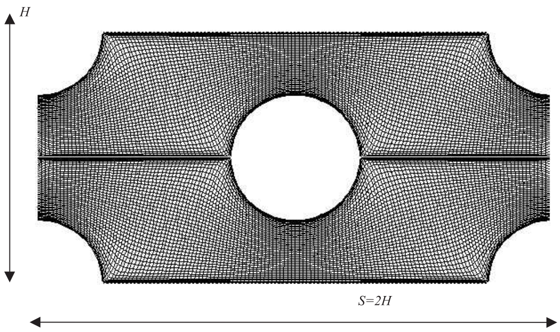

Fig. 2. Model of REV periodic cell and elliptically generated grid.

The numerical method SIMPLE was employed for relaxing the mean and turbulence equations within the domain [25]. The dimensions of the periodic cell for the cases considered in this work were $H=0.1$ m, $S=2H$, $D=0.03$ m ($\phi=0.8$), 0.05 m ($\phi=0.6$) and 0.06 m ($\phi=0.4$). The solutions were grid-independent and all normalized residuals were brought down to 10^{-5}. Also, relaxation parameters for all variables ($\mathbf{u}, p, k, \varepsilon$) were kept equal to 0.8. A summary of all relevant parameters is presented in Table 1. Overall pressure drop for a large range of Re_H compared with results of Kuwahara et al. [12] is presented in Fig. 3. Results obtained for laminar flow and for both turbulence models used (low and high Re forms) agreed well with published numerical data. Slight lower pressure drop values shown in Fig. 3 for large values of Re_H could be due to the fact that in Kuwahara et al. [12] the porous structure was simulated by an infinite array of square rods instead of cylinders as here analyzed. Flow past cylinders presents smother flow pattern, eventually implying in lower pressure drag for larger Reynolds numbers.

7. Constant c_k for the macroscopic model

The constant c_k introduced in Eq. (74) must be determined for closure of the macroscopic mathematical model proposed. In this work, a methodology was devised in order to obtain such value. Accordingly, the need of computing the fine flow properties in order to

Table 1
Parameters for microscopic computations (velocities in m/s)

Re_H	$\phi = 0.4$		$\phi = 0.6$		$\phi = 0.8$		Turbulence model
	$\bar{\mathbf{u}}_D$	$\langle\bar{\mathbf{u}}\rangle^i$	$\bar{\mathbf{u}}_D$	$\langle\bar{\mathbf{u}}\rangle^i$	$\bar{\mathbf{u}}_D$	$\langle\bar{\mathbf{u}}\rangle^i$	
1.20E+01	1.80E−04	4.50E−04	1.79E−04	2.99E−04	1.79E−04	2.24E−04	Laminar
1.20E+04	1.80E−01	4.50E−01	1.79E−01	2.99E−01	1.79E−01	2.24E−01	Low *Re*
1.20E+05	1.80E+00	4.50E+00	1.79E+00	2.99E+00	1.79E+00	2.24E+00	Low *Re*
1.20E+05	1.80E+00	4.50E+00	1.79E+00	2.99E+00	1.79E+00	2.24E+00	High *Re*
1.20E+06	1.80E+01	4.50E+01	1.79E+01	2.99E+01	1.79E+01	2.24E+01	High *Re*

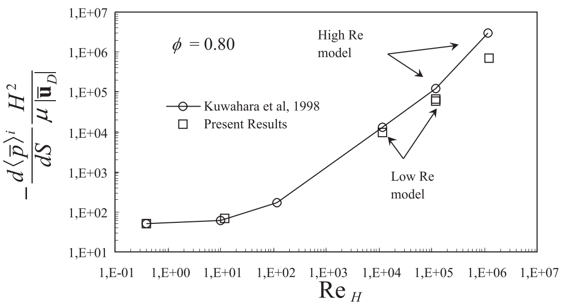

Fig. 3. Overall pressure drop as a function of Re_H for the cell of Fig. 2.

obtain the volume-integrated quantities has motivated the development of adequate numerical tools. As mentioned, those calculations were needed for adjusting the model and considered either the high *Re* k–ε closure [26] as well as the low Reynolds version of it [27]. Heat transfer analysis was also the subject of additional research [28]. One of the outcomes of this development was the ability to treat hybrid computational domains with a single numerical tool [29,30]. Major results from such methodology are presented below.

For macroscopic fully developed uni-dimensional flow in isotropic and homogeneous media the limiting values for $\langle k\rangle^i$ and $\langle\varepsilon\rangle^i$ in the additional terms introduced in equations and are given the values k_ϕ and ε_ϕ, respectively. In this limiting condition, Eqs. (75) and (85) will reduce to

$$\langle\varepsilon\rangle^i = c_k \frac{\langle k\rangle^i|\bar{\mathbf{u}}_D|}{\sqrt{K}} \quad \text{and} \quad \frac{\langle\varepsilon\rangle^{i^2}}{\langle k\rangle^i} = c_k \frac{\langle\varepsilon\rangle^i|\bar{\mathbf{u}}_D|}{\sqrt{K}}. \tag{86}$$

Using then the limiting cases k_ϕ and ε_ϕ, both Eqs. (86) can be combined into the non-dimensional form,

$$\frac{\varepsilon_\phi\sqrt{K}}{\left|\bar{\mathbf{u}}_D\right|^3} = c_k \frac{k_\phi}{\left|\bar{\mathbf{u}}_D\right|^2}. \tag{87}$$

The permeability used in Eq. (87) was calculated by solving the flow equations inside the grid of Fig. 2, for the Darcy regime (creeping flow, $Re_H < 1$), and also by using the expression proposed in [12] of the form

$$K = \frac{\phi^3}{144(1-\phi)^2}D^2. \tag{88}$$

The first value was given the name K_{calc} and was obtained by calculating the macroscopic pressure gradient across the periodic cell and applying it to the standard Darcy law of flow. For different porosity ϕ, a comparison with values from Eq. (88) is presented in Table 2.

In order to obtain c_k, the microscopic computations described above for different porosity and Re_H were used to calculate the corresponding limiting values k_ϕ and ε_ϕ (see Table 1). Once these intrinsic values were obtained, they were plugged into Eq. (87) with the permeability calculated by (88) (see Table 2). The value of c_k equal to 0.28 was found by noting the collapse of all data into the straight line shown in Fig. 4.

8. Macroscopic model results

Once the constant c_k is determined, calculation using the macroscopic turbulence model here presented can be achieved. A test case consisting in calculating a domain of length $10H$ starting with a pre-selected initial conditions greater than the final asymptotic values is now carried out. Similar test results were reported by Nakayama and Kuwahara [15] being the values at entrance $\langle k\rangle^i = 10k_\phi$ and $\langle\varepsilon\rangle^i = 30\varepsilon_\phi$. Figs. 5 and 6 show results of

Table 2
Permeability for periodic cell of Fig. 2

ϕ	$\bar{\mathbf{u}}_D$ (m/s)	Re_H	K (Eq. (88), m^2)	K_{calc} (m^2)
0.80	5.76E−06	3.88E−01	2.34E−04	1.97E−04
0.60	5.54E−06	3.70E−01	4.84E−05	4.45E−05
0.40	5.30E−06	3.54E−01	9.44E−06	5.22E−06

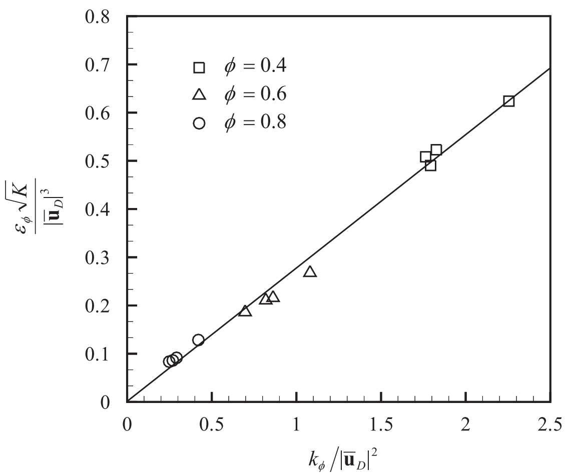

Fig. 4. Determination of value for c_k using data for different porosity end Reynolds number.

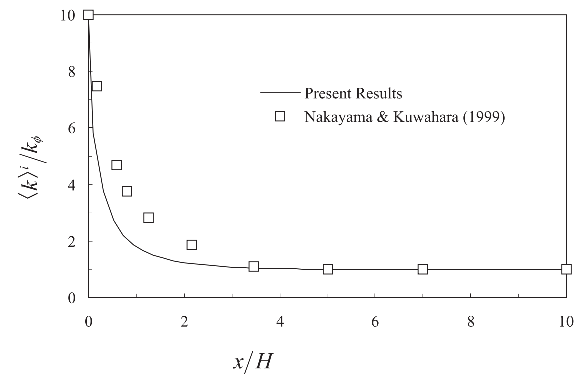

Fig. 5. Development of macroscopic TKE.

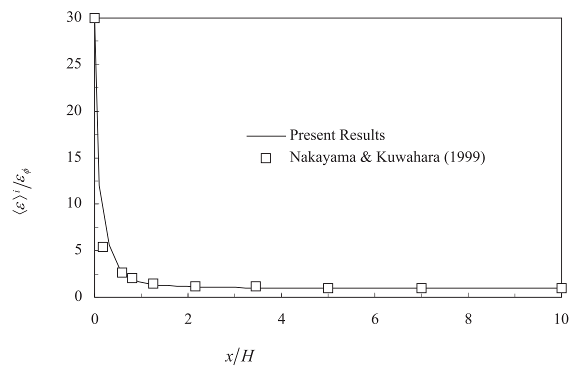

Fig. 6. Development of non-dimensional dissipation rate.

the macroscopic model for $\langle k \rangle^i$ and $\langle \varepsilon \rangle^i$ along the flow development. Results are compared with computation presented in [15]. It is interesting to observe that in [15] the medium was modeled as an array of square rods which could account for the larger values for $\langle k \rangle^i$ during flow development in contrast with the somewhat "smoother" flow around circular cylinder. In this former case, generation of turbulence past sharp corners along the flow would tend to increase overall levels of turbulent kinetic energy. Nevertheless, axial decay is nearly the same in both cases giving support to the model here presented.

9. Conclusions

The two point of views presented in the literature with proposals for turbulence models in porous media were observed. It was proved that for the mean flow field, the order of application of both time and volume averaging operators is immaterial in regard to the final set of equations obtained.

A proposal for a macroscopic turbulence model based on volume integration of clear fluid equations was carried out. Additional terms accounting for the presence of the solid matrix were introduced along with an additional constant to be determined.

The methodology followed for determining the introduced constant c_k consisted in solving the clear fluid equations within a periodic cell. Integrated parameters considering different porosity and Reynolds number were used to establish the value of the proposed constant. Simulation of the flow in the entrance region of homogeneous isotropic porous medium was compared with numerical results in the recent literature and agreed well with published numerical data.

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References

[1] H. Darcy, Les Fontaines Publiques de la Vile de Dijon, Victor Dalmond, Paris, 1856.

[2] P. Forchheimer, Wasserbewegung durch boden, Z. Ver. Deutsch. Ing. 45 (1901) 1782–1788.

[3] H.C. Brinkman, A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles, Appl. Sci. Res. A 1 (1947) 27–34.

[4] J.C. Ward, Turbulent flow in porous media, J. Hydraul. Div. ASCE 90 (HY5) (1964) 1–12.

[5] S. Whitaker, Advances in theory of fluid motion in porous media, Ind. Eng. Chem. 61 (1969) 14–28.

[6] J. Bear, Dynamics of Fluids in Porous Media, American Elsevier, New York, 1972.

[7] K. Vafai, C.L. Tien, Boundary and inertia effects on flow and heat transfer in porous media, Int. J. Heat Mass Transfer 24 (1981) 195–203.

[8] K. Lee, J.R. Howell, Forced convective and radiative transfer within a highly porous layer exposed to a turbulent external flow field, in: Proceedings of the 1987 ASME–JSME Thermal Engineering Joint Conference, Honolulu, Hawai, vol. 2, 22–27 March 1987, pp. 377–386.
[9] H. Wang, E.S. Takle, Boundary-layer flow and turbulence near porous obstacles, Boundary Layer Meteorology 74 (1995) 73–88.
[10] B.V. Antohe, J.L. Lage, A general two-equation macroscopic turbulence model for incompressible flow in porous media, Int. J. Heat Mass Transfer 40 (1997) 3013–3024.
[11] T. Masuoka, Y. Takatsu, Turbulence model for flow through porous media, Int. J. Heat Mass Transfer 39 (1996) 2803–2809.
[12] F. Kuwahara, Y. Kameyama, S. Yamashita, A. Nakayama, Numerical modeling of turbulent flow in porous media using a spatially periodic array, J. Porous Media 1 (1998) 47–55.
[13] F. Kuwahara, A. Nakayama, Numerical modeling of non-Darcy convective flow in a porous medium, in: Proceedings of the 11th International Heat Transfer Conference, Kyongyu, Korea, vol. 4, 23–28 August 1998, pp. 411–416.
[14] Y. Takatsu, T. Masuoka, Turbulent phenomena in flow through porous media, J. Porous Media 3 (1998) 243–251.
[15] A. Nakayama, F. Kuwahara, A macroscopic turbulence model for flow in a porous medium, ASME J. Fluids Eng. 121 (1999) 427–433.
[16] V.S. Travkin, K. Hu, I. Catton, Turbulent kinetic energy and dissipation rate equation models for momentum transport in porous media, in: Proceedings of the Third ASME/JSME Joint Fluids Engineering Conference (on CD-ROM), Paper FEDSM99-7275, San Francisco, California, 18–23 July 1999.
[17] W.G. Gray, P.C.Y. Lee, On the theorems for local volume averaging of multiphase system, Int. J. Multiphase Flow 3 (1977) 333–340.
[18] J.C. Slattery, Flow of viscoelastic fluids through porous media, AIChEJ 13 (1967) 1066–1071.
[19] C.T. Hsu, P. Cheng, Thermal dispersion in a porous medium, Int. J. Heat Mass Transfer 33 (1990) 1587–1597.
[20] M.H.J. Pedras, M.J.S. de Lemos, On the definition of turbulent kinetic energy for flow in porous media, Int. Commun. Heat Mass Transfer 27 (2) (2000) 211–220.
[21] M.H.J. Pedras, M.J.S. de Lemos, On volume and time averaging of transport equations for turbulent flow in porous media, in: Proceedings of the Third ASME/JSME Joint Fluids Engineering Conference (on CD-ROM), Paper FEDSM99-7273, San Francisco, CA, 18–23 July 1999.
[22] J.O. Hinze, Turbulence, McGraw-Hill, New York, 1959.
[23] H. Tennekes, J.L. Lumley, A First Course in Turbulence, MIT Press, Massachusetts, 1972.
[24] K. Abe, Y. Nagano, T. Kondoh, An improved k–ε model for prediction of turbulent flows with separation and reattachment, Trans. JSME Ser. B 58 (554) (1992) 3003–3010.
[25] S.V. Patankar, Numerical Heat Transfer and Fluid Flow, McGraw-Hill, New York, 1980.
[26] F.D. Rocamora Jr., M.J.S. de Lemos, Numerical solution of turbulent flow in porous media using a spatially periodic aArray and the k–ε model, in: Proceedings of the ENCIT-98 – 7th. Braz. Cong. Eng. Th. Sci., Rio de Janeiro, RJ, Brazil, vol. 2, 3–6 November 1998, pp. 1265–1271.
[27] M.H.J. Pedras, M.J.S. de Lemos, Numerical solution of turbulent flow in porous media using a spatially periodic cell and the low Reynolds k–ϵ model, to be presented at CONEM2000 – National Mechanical Engineering Congress, Natal, Rio Grande do Norte, Brazil, 7–11 August 2000 (in Portuguese).
[28] F.D. Rocamora Jr., M.J.S. de Lemos, Simulation of turbulent heat transfer in porous media using a spatially periodic cell and the k–ε model, in: Proceedings of the 15th Braz. Congr. Mech. Eng. (on CD-ROM), ISBN: 85-85769-03-3, Águas de Lindóia, São Paulo, Brazil, 22–26 November 1999.
[29] F.D. Rocamora Jr., M.J.S. de Lemos, Prediction of velocity and temperature profiles for hybrid porous medium-clean fluid domains, to be presented at CONEM2000 – National Mechanical Engineering Congress, Natal, Rio Grande do Norte, Brazil, 7–11 August 2000.
[30] F.D. Rocamora Jr., M.J.S. de Lemos, Laminar recirculating flow and heat transfer in hybrid porous medium-clear fluid computational domains, to be presented at 34th ASME – National Heat Transfer Conference, Paper NHTC2000-12317, Pittsburgh, PA, 20–22 August 2000.